TRANSIENT ANALYSIS OF A THERMOELECTRIC COOLING ELEMENT

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CERTIFICATE

Certified that the thesis entitled " **TRANSIENT ANALYSIS OF A THERMOELECTRIC COOLING ELEMENT** " by Mr. Sameer Joshi has been carried out under my supervision and that this work has not been submitted elsewhere for the award of a degree.

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ABSTRACT

The thesis presents a theoretical investigation carried out to study the transient behaviour of an element of a thermoelectric cooling device. A mathematical model of a simple thermoelectric device has been developed and the resulting equations solved under various boundary conditions. The hot and cold junction temperatures have been plotted against time for the various conditions. In order to carry out a comparative evaluation of the influence of the various parameters viz. current and thermal load, a wide range of values have been considered for each case.

It is found that Peltier cooling at the cold junction occurs most rapidly, followed by heat conduction from adjacent sections of the thermoelements. The process of the transfer of the Joulean heat generated within the volume of the thermoelements takes the longest time, but exercises an overriding influence, especially at higher current values.

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SAMEER JOSHI

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NOMENCLATURE

x	position coordinate, m
t	time coordinate, s
T_1	$= T_1(x,t)$, temperature at any point and at any time in element 1, K
T_2	$= T_2(x,t)$, temperature at any point and at any time in element 2, K
T_o	temperature of the junction at $x = 0$, K
T_l	temperature of the junction at $x = l$, K
P_o	rate at which heat enters the junction at $x = 0$, W
P_l	rate at which heat enters the junction at $x = l$, W
I	current, A
V	voltage, V
a_1	cross sectional area of element 1, m^2
a_2	cross sectional area of element 2, m^2
l	length of the elements, m
ρ_1	electrical resistivity of element 1, Ω-m
ρ_2	electrical resistivity of element 2, Ω-m
k_1	thermal conductivity of element 1, W/m-K
k_2	thermal conductivity of element 2, W/m-K
c_1	specific heat of element 1, J/m^3-K
c_2	specific heat of element 2, J/m^3-K
α_1	Seebeck coefficient for element 1, V/K
α_2	Seebeck coefficient for element 2, V/K

π_o	Peltier coefficient of the junction at x = 0, V
π_1	Peltier coefficient of the junction at x = 1, V
τ_1	Thomson coefficient for element 1, V/K
τ_2	Thomson coefficient for element 2, V/K
τ	net Thomson coefficient for the couple, V/K

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CHAPTER - 1

INTRODUCTION

In 1821 Seebeck[1], reported some experiments to the Prussian Academy of Sciences, in which he had discovered thermoelectric currents arising in a closed circuit made up of different conductors kept at different junction temperatures. Through extensive experimentation Seebeck was also able to arrange a great variety of solid and liquid metals and their alloys in more or less the thermoelectric series that is recognized today.

Twelve years later Peltier[2], a French watchmaker, published some results which showed that he had discovered a secon thermoelectric effect. When a current is passed through a junctio between two different conductors there is absorption or generatio of heat depending on the direction of the current. This effect i superimposed upon, but quite different from, the Joule resistanc heating effect usually associated with the passage of an electri current. Peltier, like Seebeck, did not understand the tru significance of his results, but in 1838, Lenz demonstrated tha water could be frozen at a bismuth-antimony junction by th passage of a current; on reversing the current the ice could b melted.

Thomson[3], later Lord Kelvin, realized that a relati should exist between the Seebeck and Peltier effects and proceede to derive this relation from thermodynamical arguments. This l

him to the conclusion that there must be a third thermoelectric effect now called the Thomson effect; this is a heating or cooling effect in a homogeneous conductor when an electric current passes in the direction of a temperature gradient.

The basic theory of thermoelectricity was first derived satisfactorily by Altenkirch[4,5], in 1909 and 1911. He showed that for both thermoelectric power generation and cooling, materials were required with high thermoelectric coefficients, high electrical conductivities to minimize Joule heating and low thermal conductivities to reduce heat transfer losses. However it was quite a different matter knowing the favourable properties and obtaining materials embodying them, and so long as metallic thermocouples were employed no real progress was made. It is only since semiconductor thermocouples have been developed that reasonably efficient thermoelectric devices have become possible.

1.1 Steady State versus Transient Behaviour :

Steady State Behaviour : The steady state behaviour refers to the operation of the thermoelectric cooler under static conditions of heat load, current, ambient temperature etc. The steady state behaviour of a thermoelectric device has been extensively studied and the results can be obtained from any standard text on the subject[6,7,8]. In particular the effect of various thermoelectric parameters on the performance of a thermoelectric device is given by the *figure of merit*, z, which is defined as :

$$z = \alpha^2 . \frac{\sigma}{k}$$

where α is the Seebeck coefficient (in V/K), of the junction and is equal to the difference between the two absolute Seebeck coefficients of the materials that make up the junction.

σ is the electrical conductivity in $(\Omega\text{-m})^{-1}$

k is the thermal conductivity in W/m-K

Under steady state operating conditions the coefficient of performance and the maximum temperature drop depend only on the value of z, which thus describes the quality of the thermoelectric refrigerator element.

Transient Behaviour : The transient behaviour refers to the operation of the device in the initial period, before steady state is reached. There is at present very little information available about the behaviour of thermoelectric devices under non-steady state conditions. This problem is much more difficult than the steady state one as it requires the solution of partial differential equations that are at best linear in temperature but non-linear in current. Further these equations are subject to boundary conditions that include temperature-current product type non-linearities.

CHAPTER - 2

THEORY OF THERMOELECTRIC COOLING

2.1 Interaction between Heat Flow and Electric Current [9]:

Generally the flow of heat and that of electric current are considered as isolated phenomena. It is often overlooked that the two are actually closely interrelated and hence represent only two limiting cases of a more general formulation.

Consider the situation depicted in Fig. 2.1 :

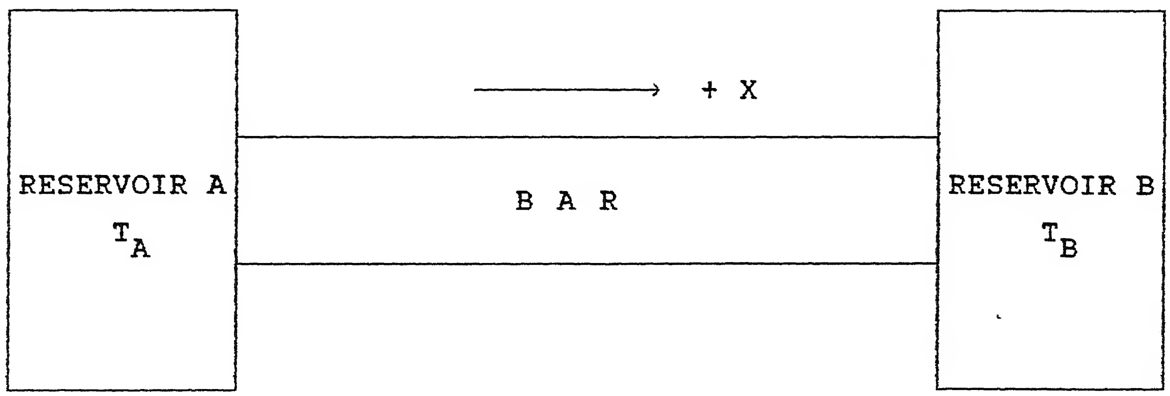


Fig. 2.1. Interdependence of Heat Flux and Electric Current

The two ends of a metallic bar are in contact with two heat reservoirs maintained at different temperatures. If $T_A > T_B$, then heat is transferred by conduction from reservoir A to reservoir B In addition, any mobile charge carriers present, possess a higher average thermal energy at the hot end A as compared to the coo end B . Consequently the average velocity component along th

positive x direction of carriers at A exceeds the average velocity component along the negative x direction of the carriers at B . This imbalance leads to a net flux of mobile particles from A to B until the charges accumulating at B and the resulting deficit at A give rise to an internal electric field which ultimately stops any further net transfer of carriers. Under steady state conditions the concentration of mobile carriers at the cold end exceeds that at the hot end, and a difference of potential is developed across the bar. During the establishment of the steady state, an electric current accompanies the normal heat flux.

Now suppose an external source causes the potential difference between the ends of the bar to be altered in such a way as to induce a flow of carriers from left to right in the positive x direction. Carriers originally at temperature T_A find themselves displaced to cooler surroundings, and they discharge their excess kinetic energy by collision with the lattice. This process assists the normal conduction of heat from the hot to the cold junction. If however the potential gradient is so altered as to cause a flow of carriers along the negative x direction, then particles at temperature T_B accumulate at the hotter end, thereby altering the heat flux which would be observed in the absence of a current.

2.2 Origin of Thermoelectric Cooling in a Semiconductor[6,10] :

A semiconductor may be defined as an electrical conductor with a finite forbidden energy gap between the valence and the conduction bands. In an insulator the valence band is completely

full and the conduction band is empty of electrons. Quantum mechanical considerations show that, in these circumstances, no electric current can flow. In an *intrinsic* semiconductor, charge carriers are produced by thermal excitation of electrons from the valence band to the conduction band, across the forbidden energy gap. In an *extrinsic* semiconductor the presence of certain impurities (donors) may provide electrons in levels which are close enough to the conduction band for them to be excited into the latter at relatively low temperatures. Alternatively, other impurities (acceptors) may provide vacant levels, close to the valence band, into which electrons from the latter may pass. The vacant spaces in the otherwise full valence band behave like free electrons with a positive charge and are usually known as positive holes. Semiconductors are called *n - type* when the conduction of electricity is primarily due to electrons, and *p - type* when conduction is mainly due to positive holes.

In an electric circuit when an electron flows from the connecting metal strip into the n - type semiconductor, it has to rise into the conduction band of the latter, absorbing an equivalent amount of potential energy in the process. This energy absorption at the junction is the origin of the Peltier cooling effect, and the Peltier coefficient is, thus, the average value of this energy change per unit charge. By convention it is negative for an n - type semiconductor and positive for a p - type material.

2.3 Thermoelectric Effects :

Three thermoelectric effects govern the behaviour of any thermoelectric device in general. These are ;

(i) The Seebeck Effect

(ii) The Peltier Effect

(iii) The Thomson Effect.

These effects are a manifestation of the direct reversible conversion of electrical energy into thermal energy and vice-versa.

2.3.1 Seebeck Effect :

If a closed circuit is made of two dissimilar metals, an electric current flows in the circuit when the two junctions are maintained at different temperatures. For a small temperature difference between the two junctions the open circuit voltage is proportional to the temperature difference :

$$\Delta V = \alpha . \Delta T$$

where ΔV = open circuit voltage developed

α = Seebeck Coefficient

ΔT = temperature difference between the two junctions.

For a small temperature difference ΔT, this can be written as ;

$$\alpha = \lim_{\Delta T \to 0} \left(\frac{\Delta V}{\Delta T} \right) = \frac{dV}{dT} \qquad \ldots(2.1)$$

which gives us the definition of the Seebeck Coefficient.

The Seebeck Coefficient depends upon the choice of the two materials and cannot be attributed to either material alone. In

practice, the absolute Seebeck coefficient of a material is determined with respect to a material such as lead in which the Seebeck coefficient is negligible. The Seebeck coefficient of a thermocouple made of two different materials is the difference between the absolute Seebeck coefficients of the two materials.

2.3.2 Peltier Effect :

When a direct current is passed through a junction of two dissimilar metals, the junction becomes either hot or cold. The rate of liberation or absorption of heat is proportional to the current and is given by:

$$Q = \Pi . I \qquad \ldots(2.2)$$

where Q = rate of heat liberated or absorbed

Π = Peltier Coefficient

I = the direct current in the circuit.

As in the case of the Seebeck coefficient the Peltier coefficient too cannot be ascribed to either material alone but rather to the junction as a whole.

2.3.3 Thomson Effect :

Unlike the Seebeck and Peltier effects which take place only at the junctions, the Thomson effect involves the interchange of energy with the surroundings throughout the bulk of the conductor if an electric current flows through it, while a temperature gradient exists across the conductor. The power absorbed or liberated per unit length is given by the expression ;

$$\frac{dQ}{dx} = \tau.I.(\frac{dT}{dx}) \qquad ...(2.3)$$

where $\frac{dQ}{dx}$ = power absorbed or liberated per unit length

τ = the Thomson coefficient

I = the current through the conductor

$\frac{dT}{dx}$ = temperature gradient across the conductor.

Again unlike the Seebeck and Peltier coefficients the Thomson coefficient depends solely on the properties of the single material.

The Seebeck, Peltier and Thomson effects are reversible in nature. In addition to the above three reversible effects associated with thermoelectricity, there are two other irreversible effects that take place - Jouleean Heating and Thermal Conduction from the hot junction to the cold junction. All the reversible and irreversible effects have to be taken into account in thermoelectric cooling analysis.

CHAPTER - 3

LITERATURE SURVEY AND THE PRESENT WORK

3.1 Literature Survey :

Though the principles behind thermoelectric cooling have been known for a long time, practical devices based on them could only be developed recently, after the development of new semiconductor materials. Thus thermoelectric cooling is still one of the young branches of modern technology. It has assumed an added significance in recent years because of the need for elimination of CFCs and the consequent need to develope alternative refrigeration technologies.

E. A. Kolenko [11] has described some of the earliest engineering designs of thermoelectric cooling devices, developed in the sixties at the Institute of Semiconductors of the U.S.S.R. Academy of Sciences. However these devices are all based on steady state principles only.

A. F. Ioffe [12], in his pioneering work on thermoelectricity has briefly written about the operation of thermoelectric devices in the initial stages, before steady state is reached.

L. S. Stil'bans and N. A. Fedorovich [13] analysed the operation of refrigerating thermoelectric elements in non-steady state conditions. They modelled the cold junction of the thermocouple as a lumped mass, and obtained an exponentially decreasing temperature profile when the current was passed through

the device.

A. G. Shcherbina [11] has reported on the operation of semiconductor refrigerators in the initial period before steady state conditions are reached. In his analysis he has considered a single thermoelement and has modelled the heat transfer as a volume process. First he has considered the simplest operating condition in which the temperature has been taken as constant and equal to the temperature of the ambient medium. The second case is a modification of the first one, in as much as the hot junctions are taken to be in contact with a moving liquid, the temperature of which is taken to be constant and equal to the temperature of the surrounding medium. In both the cases the final expression for the cold junction temperature shows an exponential decrease with time.

A very comprehensive analysis of the transient behaviour of thermoelectric devices has been carried out by P. E. Gray [14]. He has derived the general governing equations that describe the phenomena and has obtained solutions for various operating conditions. In his analysis Gray has assumed that each of the variables can be written as the sum of a large-signal static or quiescent term, which is not a function of time, and a small-signal transient or incremental term which is small compared to the corresponding quiescent term. The separation leads to large-signal static equations and small-signal transient equations. The latter are then Fourier analysed to obtain the transfer functions which describe the behaviour of the device under various conditions of interest.

3.2 Present Work :

In recent years thermoelectric cooling devices have been developed for use in nuclear physics, vacuum technology, metallurgy, medicine, botany, electronics and many other branches of science and technology. Many such devices require the temperature to be controlled within a very narrow range. For example a thermoelectric spot cooler may be required to control the temperature of an electronic component that dissipates heat at a variable rate. For the design and development of such devices their transient behaviour must first be investigated.

From the literature survey it is apparent that thermoelectricity and its application to refrigeration is still a relatively new field. This is especially true of the transient behaviour of such devices where the work done mainly involves gross approximations, and a rigorous analysis of the phenomena has not even been attempted. Hence the need for the present work to formulate a mathematical model of an element of a thermoelectric cooler. The two thermoelectric effects (Peltier cooling and Thomson effect) and the associated effects (Joulean heat generation and heat conduction) have been considered in their entirety, to derive the generalized governing equations for the system. The equations have been solved numerically under various boundary conditions of practical interest. Reasonable assumptions have been made only where necessary to ensure accurate results.

CHAPTER - 4

DERIVATION OF THE GENERAL GOVERNING EQUATIONS

4.1 Mathematical Model of the System :

A thermoelectric cooling element in its simplest form is a simple p-n thermoelectric couple. It consists of two homogeneous semiconductors to which electrical connections are made by metal strips that are assumed to have negligibly small specific heats, thermal and electrical resistances. The geometrical arrangement is shown schematically in Fig. 4.1 :

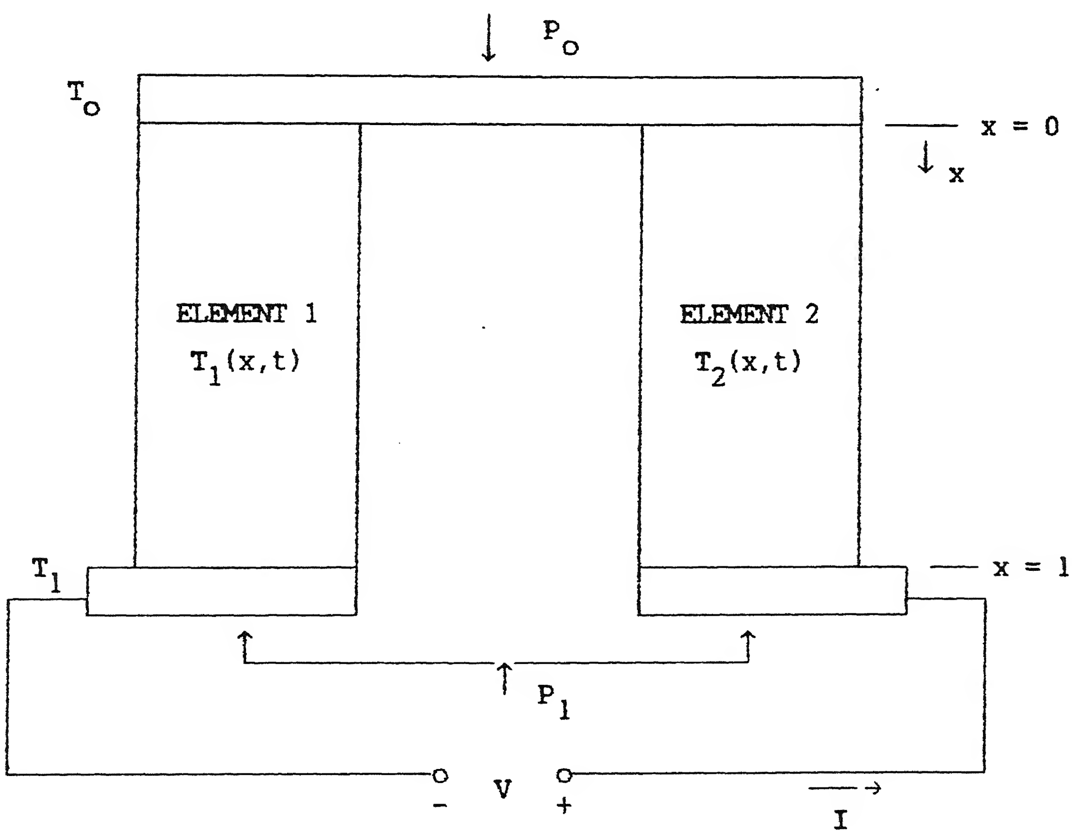


Fig. 4.1. Model of the Thermoelectric Device

The thermoelements, which are uniform in cross section are assumed to have the same length. The sides of the thermoelements are further assumed to be insulated so that the flow of heat is along the length of the thermoelements only (one dimensional flow of heat).

4.2 Sign Conventions Followed :

(i) The position coordinate, x, is measured downwards from the upper junction which is taken at x = 0.

(ii) The heat *entering* the junctions both at x = 0 and at x = 1 is taken as positive.

(iii) The current is defined as *positive* in the *positive x-direction* in element 1 and in the *negative x-direction* in element 2.

(iv) The terminal voltage is defined such that the rate at which energy *leaves* the system at the electrical terminals is given by V*I watts.

(v) A *positive* Thomson coefficient denotes *liberation* of heat if the temperature gradient and electric current are in the same direction. The net Thomson coefficient for the couple is given by:

$$\tau = \tau_1 - \tau_2 \qquad \text{...(4.1)}$$

(vi) The Peltier coefficient π_0 which is defined such that a *positive* coefficient designates *heat liberation* when the current flow is *positive* (that is from element 2 to element 1). Similarly the Peltier coefficient π_1 is defined such that a *positive* coefficient designates *heat absorption* for a *positive* current flow.

4.3 Derivation of the General Governing Equations :

The general governing equations that completely describe the behaviour of the cooling element shown in Fig. 4.1, consist of two differential equations and three energy balance equations :

(i) Two differential equations that must be satisfied by the temperature distributions in the elements.

(ii) Energy balance for each junction which relates P_0 and P_1, to the electric currents and temperatures of the two elements.

(iii) Overall energy balance equation.

4.3.1 Differential Equations for Temperature Distribution :

Consider a differential strip of element 1 , of length dx at a distance x from the origin :

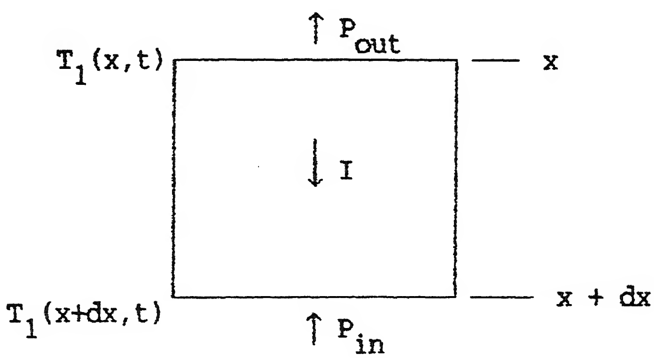


Fig. 4.2. Differential Strip of Element 1

The rate at which heat is conducted into the segment at x + dx is :

$$P_{in} = k_1.a_1.\left.\frac{\partial T_1}{\partial x}\right|_{x+dx} \qquad \ldots(4.2)$$

The rate at which heat is conducted out of the segment at x is :

$$P_{out} = k_1 . a_1 . \left.\frac{\partial T_1}{\partial x}\right|_x \qquad \ldots(4.3)$$

The rate at which Joule heating takes place in the segment is

$$P_J = (\text{current})^2 \times (\text{resistance of the segment})$$

$$= I^2 . \rho_1 . \frac{dx}{a_1} \qquad \ldots(4.4)$$

The rate at which heat is liberated due to the Thomson effect is :

$$P_\tau = I . \tau_1 . \frac{\partial T_1}{\partial x} . dx \qquad \ldots(4.5)$$

The rate at which stored energy is increasing is :

$$P_s = c_1 . a_1 . \frac{\partial T_1}{\partial x} . dx \qquad \ldots(4.6)$$

Conservation of energy demands that :

Net energy conducted into the segment + Joule heating within the segment + Heat liberated due to the Thomson effect = Rate at which the stored energy increases.

ie. $$P_{in} - P_{out} + P_J + P_\tau = P_s \qquad \ldots(4.7)$$

or $$P_{in} - P_{out} = P_s - P_J - P_\tau$$

$$\text{ie.}\quad k_1.a_1.\left.\frac{\partial T_1}{\partial x}\right|_{x+dx} - k_1.a_1.\left.\frac{\partial T_1}{\partial x}\right|_{x} = c_1.a_1.\frac{\partial T_1}{\partial x}.dx - I^2.\rho_1.\frac{dx}{a_1} - I.\tau_1.\frac{\partial T_1}{\partial x}.dx$$

$$\text{ie.}\quad \frac{\partial}{\partial x}\left(k_1.a_1.\frac{\partial T_1}{\partial x}\right).dx = c_1.a_1.\frac{\partial T_1}{\partial x}.dx - I^2.\rho_1.\frac{dx}{a_1} - I.\tau_1.\frac{\partial T_1}{\partial x}.dx$$

$$\text{ie.}\quad \frac{\partial}{\partial x}\left(k_1.a_1.\frac{\partial T_1}{\partial x}\right) + \tau_1.I.\frac{\partial T_1}{\partial x} + \frac{\rho_1}{a_1}.I^2 - c_1.a_1.\frac{\partial T_1}{\partial x} = 0 \qquad \text{...(4.8)}$$

which is the differential equation that governs the temperature distribution in element 1. The derivation of the differential equation for element 2 is identical except that the current I is in the opposite direction and so must be replaced by -I. Hence the equation for the temperature distribution in element 2 is :

$$\frac{\partial}{\partial x}\left(k_2.a_2.\frac{\partial T_2}{\partial x}\right) - \tau_2.I.\frac{\partial T_2}{\partial x} + \frac{\rho_2}{a_2}.I^2 - c_2.a_2.\frac{\partial T_2}{\partial x} = 0 \qquad \text{...(4.9)}$$

4.3.2 Junction Energy Balance :

This is obtained by recognizing that the power which enters either the upper or lower junctions is the sum of the powers that are removed from the corresponding junctions by the Peltier effect and by thermal conduction through the the two elements. Thus,

$$P_o(t) = -\Pi_o . I(t) - k_1.a_1.\left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=0} - k_2.a_2.\left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=0}$$

...(4.10)

and

$$P_l(t) = \Pi_l . I(t) + k_1.a_1.\left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=l} + k_2.a_2.\left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=l}$$

...(4.11)

Now the first Kelvin relation gives us :

$$\alpha_l . T_l = \Pi_l \qquad ...(4.12)$$

and

$$\alpha_o . T_o = \Pi_o \qquad ...(4.13)$$

and the second Kelvin relation gives us :

$$\frac{d\alpha_o}{dT_o} = \frac{\tau}{T_o} \qquad ...(4.14)$$

and

$$\frac{d\alpha_l}{dT_l} = \frac{\tau}{T_l} \qquad ...(4.15)$$

[Please see Appendix - A for the derivation of the Kelvin relations.]

Using equations (4.10) and (4.11) we get :

$$P_o(t) = -\alpha_o . T_o(t) . I(t) - k_1.a_1.\left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=0} - k_2.a_2.\left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=0}$$

...(4.16)

and

$$P_l(t) = \alpha_1 . T_1(t) . I(t) + k_1 . a_1 . \left. \frac{\partial T_1(x,t)}{\partial x} \right|_{x=l} + k_2 . a_2 . \left. \frac{\partial T_2(x,t)}{\partial x} \right|_{x=l} \qquad \ldots (4.17)$$

which represent the energy balance at the two junctions.

4.3.3 Overall Energy Balance Equation :

The difference between the net thermal power input and the electrical power output must equal the rate at which the stored energy in the system is increasing :

$$\text{ie.} \quad P_l(t) + P_o(t) - V(t).I(t) = \frac{dW(t)}{dt} \qquad \ldots (4.18)$$

where W(t) denotes the total energy stored in the system, which is simply the energy accounted for by the specific heats of the elements :

$$W(t) = a_1 . \int_0^l c_1 . T_1(x,t).dx \; + \; a_2 . \int_0^l c_2 . T_2(x,t).dx \qquad \ldots (4.19)$$

Substituting for W(t), ${}_oP$ (t) and ${}_lP$ (t) into equation 4.18 and rearranging the terms we get :

$$V = \left[\left(\alpha_l . T_l - \alpha_o . T_o \right) - \int_{T_o}^{T_l} (\tau_1 - \tau_2).dT \right]$$

$$- I . \left[\frac{1}{a_1} . \int_0^l \rho_1 .dx + \frac{1}{a_2} . \int_0^l \rho_2 .dx \right] \qquad \ldots (4.20)$$

Since the variations in heat load and consequently current are generally small all the parameters except resistivity may be assumed to be independent of temperature.

The reasons for this are :

(i) Resistivity shows the greatest variation amongst all the parameters.

(ii) Resistivity enters the equations as the coefficient of the Joule heating term in the differential equations and this term has a quadratic current dependence.

Further the resistivity can be assumed to have a linear variation with temperature [7,8] :

ie. $\rho_1 = \rho_{10} + \eta_1 . T_1$ ...(4.21)

$\rho_2 = \rho_{20} + \eta_2 . T_2$ ...(4.22)

Substituting these expressions for resistivities into equations 4.8 and 4.9 we get :

$$k_1 . a_1 . \frac{\partial^2 T_1}{\partial x^2} + \tau_1 . I . \frac{\partial T_1}{\partial x} - c_1 . a_1 . \frac{\partial T_1}{\partial t} + \frac{\eta_1}{a_1} . I^2 . T_1 + \frac{\rho_{10}}{a_1} . I^2 = 0 \quad \ldots(4.23)$$

and

$$k_2 . a_2 . \frac{\partial^2 T_2}{\partial x^2} - \tau_2 . I . \frac{\partial T_2}{\partial x} - c_2 . a_2 . \frac{\partial T_2}{\partial t} + \frac{\eta_2}{a_2} . I^2 . T_2 + \frac{\rho_{20}}{a_2} . I^2 = 0 \quad \ldots(4.24)$$

Also equation 4.20 simplifies to :

$$V = \Big[\big(\alpha_1.T_1 - \alpha_o.T_o\big) - (\tau_1 - \tau_2).(T_1 - T_o)\Big]$$
$$- I.\left[\frac{1}{a_1}.\int_0^l(\rho_{10} + \eta_1.T_1)dx + \frac{1}{a_2}.\int_0^l(\rho_{20} + \eta_2.T_2)dx\right]$$

ie. $$V = \Big[\big(\alpha_1.T_1 - \alpha_o.T_o\big) - (\tau_1 - \tau_2).(T_1 - T_o)\Big]$$
$$- I.\left[\frac{1}{a_1}(\rho_{10}.l + \eta_1.\int_0^l T_1 dx) + \frac{1}{a_2}(\rho_{20}.l + \eta_2.\int_0^l T_2 dx)\right]$$

ie. $$V = \Big[\big(\alpha_1.T_1 - \alpha_o.T_o\big) - (\tau_1 - \tau_2).(T_1 - T_o)\Big]$$
$$- I.l\left(\frac{\rho_{10}}{a_1} + \frac{\rho_{20}}{a_2}\right) - I\left[\frac{\eta_1}{a_1}.\int_0^l T_1\, dx + \frac{\eta_2}{a_2}.\int_0^l T_2\, dx\right] \qquad ...(4.25)$$

Thus the complete system equations that describe the generic thermoelectric device are :

$$k_1.a_1.\frac{\partial^2 T_1}{\partial x^2} + \tau_1.I.\frac{\partial T_1}{\partial x} - c_1.a_1.\frac{\partial T_1}{\partial t} + \frac{\eta_1}{a_1}.I^2.T_1 + \frac{\rho_{10}}{a_1}.I^2 = 0 \qquad ...(4.23)$$

$$k_2.a_2.\frac{\partial^2 T_2}{\partial x^2} - \tau_2.I.\frac{\partial T_2}{\partial x} - c_2.a_2.\frac{\partial T_2}{\partial t} + \frac{\eta_2}{a_2}.I^2.T_2 + \frac{\rho_{20}}{a_2}.I^2 = 0 \qquad ...(4.24)$$

$$P_o(t) = -\alpha_o . T_o(t) . I(t) - k_1 . a_1 . \left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=0} - k_2 . a_2 . \left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=0}$$

...(4.16)

$$P_l(t) = \alpha_1 . T_l(t) . I(t) + k_1 . a_1 . \left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=l} + k_2 . a_2 . \left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=l}$$

...(4.17)

and lastly,

$$V = \left[\left(\alpha_1 . T_l - \alpha_o . T_o \right) - (\tau_1 - \tau_2) . (T_l - T_o)\right]$$

$$- I . l\left(\frac{\rho_{10}}{a_1} + \frac{\rho_{20}}{a_2}\right) - I\left[\frac{\eta_1}{a_1} . \int_0^l T_1 \, dx + \frac{\eta_2}{a_2} . \int_0^l T_2 \, dx \right]$$

...(4.25)

4.4 Alternative Expression For The Heat Rejected at the Hot Junction :

The rate at which heat leaves the junction at x = l is also given by :

Rate at which heat leaves at x = l	=	Rate at which heat enters at x = 0	+	Rate at which Joule heat is generated	-	Rate at which heat is stored

and following the proper sign convention we have :

$$- P_l(t) = P_o(t) - V . I - \frac{\partial W(t)}{\partial t}$$

$$\text{ie.}\quad -P_1(t) = -\alpha_o.T_o.I - k_1.a_1.\left.\frac{\partial T_1}{\partial x}\right|_{x=0} - k_2.a_2.\left.\frac{\partial T_2}{\partial x}\right|_{x=0}$$

$$-I\left[\left(\alpha_1.T_1 - \alpha_o.T_o\right) - \int_{T_o}^{T_1}(\tau_1 - \tau_2)dT\right] + I^2.l\left(\frac{\rho_{10}}{a_1} + \frac{\rho_{20}}{a_2}\right)$$

$$+ I^2\left[\frac{\eta_1}{a_1}\int_0^l T_1 dx + \frac{\eta_2}{a_2}\int_0^l T_2 dx\right] - \frac{\partial}{\partial t}\left[a_1\int_0^l c_1.T_1 dx + a_2\int_0^l c_2.T_2 dx\right]$$

$$\text{or}\quad P_1(t) = -\alpha_1.T_1.I + k_1.a_1.\left.\frac{\partial T_1}{\partial x}\right|_{x=0} + k_2.a_2.\left.\frac{\partial T_2}{\partial x}\right|_{x=0}$$

$$-I\left[(\tau_1 - \tau_2).(T_1 - T_o)\right] - I^2.l\left(\frac{\rho_{10}}{a_1} + \frac{\rho_{20}}{a_2}\right)$$

$$-I^2\left[\frac{\eta_1}{a_1}\int_0^l T_1 dx + \frac{\eta_2}{a_2}\int_0^l T_2 dx\right] + \frac{\partial}{\partial t}\left[a_1\int_0^l c_1.T_1\, dx + a_2\int_0^l c_2.T_2\, dx\right]$$

...(4.26)

4.5 Boundary and Initial Conditions :

The temperature profile in the two thermoelements is determined by equations (4.16), (4.17), (4.23) and (4.24), subject to an initial condition and four boundary conditions.

The initial condition is given by the fact that the entire system is at the ambient temperature initially when the current flow is nil. Thus :

$T_1 = T_2 = T_A$ for all x when $t \leq 0$

where T_A is the ambient temperature.

Two boundary conditions follow from the physical requirement that the temperatures at each of the two junctions must be the same for both the thermoelements.therefore we have :

$T_1 = T_2$ at $x = 0$ and at $x = 1$

The other two boundary conditions are specified by the thermal load and current flow through the couple. The different situations considered in this work are as outlined below :

(i) In the first case the effect of a change in the control current, on the junction temperatures is analysed. The power input at the cold junction, $P_0(t)$, is taken to be constant while the power rejected at the hot junction, $P_1(t)$, is evaluated from Eq.4.26. Two cases are have been considered under the imposition of these thermal loads, namely :

(a) there is a step change in the current, and

(b) the current varies sinusoidally.

(ii) In the second situation considered the current flow is kept constant while the thermal load is varied, to study its effect on the junction temperatures. Again two cases have been considered :

(a) there is a step change in the power input, $P_0(t)$, and

(b) the power input varies sinusoidally.

The power rejected at the hot junction, $P_1(t)$, is evaluated from equation 4.26 in each case.

The discretisation of the differential equations and the method for their solution is given in Appendix-B. The discussion of the results follows in the next chapter.

CHAPTER - 5

RESULTS, DISCUSSIONS AND CONCLUSIONS

5.1 Thermoelectric Parameters of the Device :

The thermoelectric device that has been analysed in this work is a p-n couple of *bismuth telluride*. The p-type element is the pure crystal, while the n-type element is bismuth telluride doped with 5×10^{-4} percent of iodine by weight.

The pertinent thermoelectric properties of the elements as obtained from the various data plots of bismuth telluride [14,15,16,17] are given below :

$k_1 = 2.4$ W/mK

$k_2 = 2.2$ W/mK

$c_1 = 1.23 \times 10^6$ J/m^{3}K

$c_2 = 1.23 \times 10^6$ J/m^{3}K

In the expression for resistivities :

$$\rho_1 = \rho_{10} + \eta_1 . T_1 \qquad \text{...(4.21)}$$

and

$$\rho_2 = \rho_{20} + \eta_2 . T_2 \qquad \text{...(4.22)}$$

we have :

$\rho_{10} = 1.3 \times 10^{-5}$ ohm-m

$\rho_{20} = 1.3 \times 10^{-5}$ ohm-m

$\eta_1 = 0.814 \times 10^{-5}$ ohm/K

$\eta_2 = 0.814 \times 10^{-5}$ ohm/K

The geometrical parameters of the device are taken to be :

$a_1 = 0.836 \ \ cm^2$

$a_2 = 0.836 \ \ cm^2$

$l = 2.0 \ \ cm$

5.2 Results and Discussions :

CASE - i(a) : Step change in the current, with the power input at the cold junction kept constant at 0.5 W.

Thirteen values of the step current are considered from 0.5 A to 10 A, to illustrate how the cold junction and hot junction temperatures vary with increasing current. The *incremental* cold and hot junction temperatures, ie. $(T_o - T_A)$ and $(T_1 - T_A)$, where T_A is the ambient (initial) temperature, have been plotted against time for the various current values in Figs. 5.1 through 5.11.

Figs. 5.1 through 5.4 are for small step current values of 0.5 A, 0.75 A, 1.0 A and 1.5 A respectively. Initially the hot junction temperature is seen to increase while the cold junction cools down rapidly. The hot junction temperature reaches a peak and then decreases to a final steady state value.

Two features of the plots are worth noting at this stage. Firstly, the initial temperature drop in case of the cold junction and temperature rise in case of the hot junction, respectively, keeps increasing as the current increases. Secondly the final steady state value of the incremental cold junction temperature decreases while that of the hot junction increases as the current is increased.

The general trends depicted by the plots show how the device responds transiently to the three effects that occur when the current is switched on, namely the thermoelectric effects (Peltier and Thomson cooling), Joulean heat generation and heat conduction. The first observation is explained by the fact that the Peltier cooling effect, which is proportional to the first power of the current, is a surface phenomenon and has a very low inertia, whereas the process of transfer of Joulean heat, generated within the volume of the thermoelement, to the cold junction, is characterized by a much higher inertia. The rate of absorption of Thomson heat, and its effect on the cold junction temperature, is same as that of the Joulean heat generation. Thus it serves only to modify the Joulean heat, and since its effect is very small it will not be considered explicitly henceforth.

The cold junction therefore cools down rapidly at first as a result of Peltier cooling, but the Joulean heat generation and heat conduction from the hotter neighbouring areas, soon arrest any further cooling of the cold junction till it reaches a final steady state value. The hot junction temperature shoots up initially due to Peltier heating but heat conduction to the neighbouring layers soon causes the temperature to drop. The Joulean heat generation further modifies the hot junction temperature which ultimately approaches a final steady state value.

The steady state temperatures for step currents from 0.5 A to 1.5 A, are given in Table 5.1. The increasing difference, between the hot and the cold junction temperatures, shows how the Peltier cooling and heating increases as the current increases. However,

this is not the complete picture as the Joulean heat generation and heat conduction also increase with increased current values. Their effects will be evident when we consider higher current flows through the device.

Figs. 5.5 to 5.13 show the behaviour of the thermoelectric junction for step currents of 2.0 to 10.0 A. The cold junction temperature now shows a very interesting trend. It decreases sharply at first, as was seen earlier, but this rapid decrease in temperature is soon checked, so that it reaches a minimum value and then it *increases* till it reaches a final steady state value much above the minimum. The minimum temperature reached decreases, while the final steady state value increases as the current increases.

The hot junction temperatures for step currents of 2.0 A to 10.0 A (Figs.5.5 to 5.13)show a similar trend to Figs.5.1 to 5.4. The steady state temperature increases rapidly as the current increases.

These trends show very clearly the transient action of the different effects on the thermoelectric device. The Peltier effect occurs most rapidly (almost instantaneously) and so the cold junction temperature decreases rapidly as soon as the current is switched on. However, heat conduction to the adjacent regions arrests the rapid decrease in temperature . Joulean heat generation and its transfer to the junction occurs last of all, but at higher current values, this is clearly the overriding factor. Thus the temperature of the cold junction soon starts increasing and for currents of 4.0 A and more the steady state temperature is actually higher than the initial temperature which

means that the device would be quite ineffective as a thermoelectric cooler at steady state.

The hot junction temperatures too illustrate the transient action of the three effects clearly. Thus the hot junction temperature shoots up initially due to the Peltier heating. Heat conduction to the adjacent layers soon follows and is shown most clearly in Figs.5.1 to 5.4 where there is a small but clearly defined temperature drop before Joulean heating causes the temperature to increase again till a final steady state value is reached. The final steady state temperature increases rapidly as the current increases since the Joulean effect depends on the square of the current. Also the steady state is attained after a longer time interval, when the current increases, which further shows that Joule heat generation and its transfer have a much higher inertia.

The minimum cold junction temperature and the final steady state temperatures of both the hot and the cold junctions, for currents of 2.0 A to 10.0 A are also given in Table 5.1. The minimum cold junction temperature decreases almost linearly as the step current increases, since it represents mainly Peltier cooling. However the final steady state values at both the hot and the cold junctions increase rapidly as the current increases. since it depends mainly on Joulean heating, which in turn has a square current dependence.

The cold junction temperatures for the various step currents have been plotted in Fig.5.14. The plots clearly illustrate the various trends discussed above. Similarly the hot junction temperatures have been plotted in Fig.5.15 to enable a comparison,

of the response of the hot junction, to increasing step current values.

In order to illustrate the transient behaviour of the element for increasing current values, the incremental temperatures at various instants of time have been plotted against current in Figs. 5.16 and 5.17. Figs.5.16(a) to (l) are for the incremental cold junction temperatures. Figs. 5.16(a) to (e) show that initially the temperature decreases with an increase in the step current. As time increases the temperature at first decreases with current till it reaches a minimum value at a particular current. For currents larger than this optimum value the cold junction temperature shows a rapidly increasing trend as depicted in Figs. 5.16(f) to (l).

The incremental hot junction temperatures have been shown plotted against current, at various instants of time, in Figs. 5.17(a) to (l). The hot junction temperature always increases with an increase in the step current. Initially the increase in temperature with current is less pronounced as it depends on the Peltier effect alone. As time increases the additional heating due to the Joulean effect causes the hot junction temperature to increase rapidly with the increasing current.

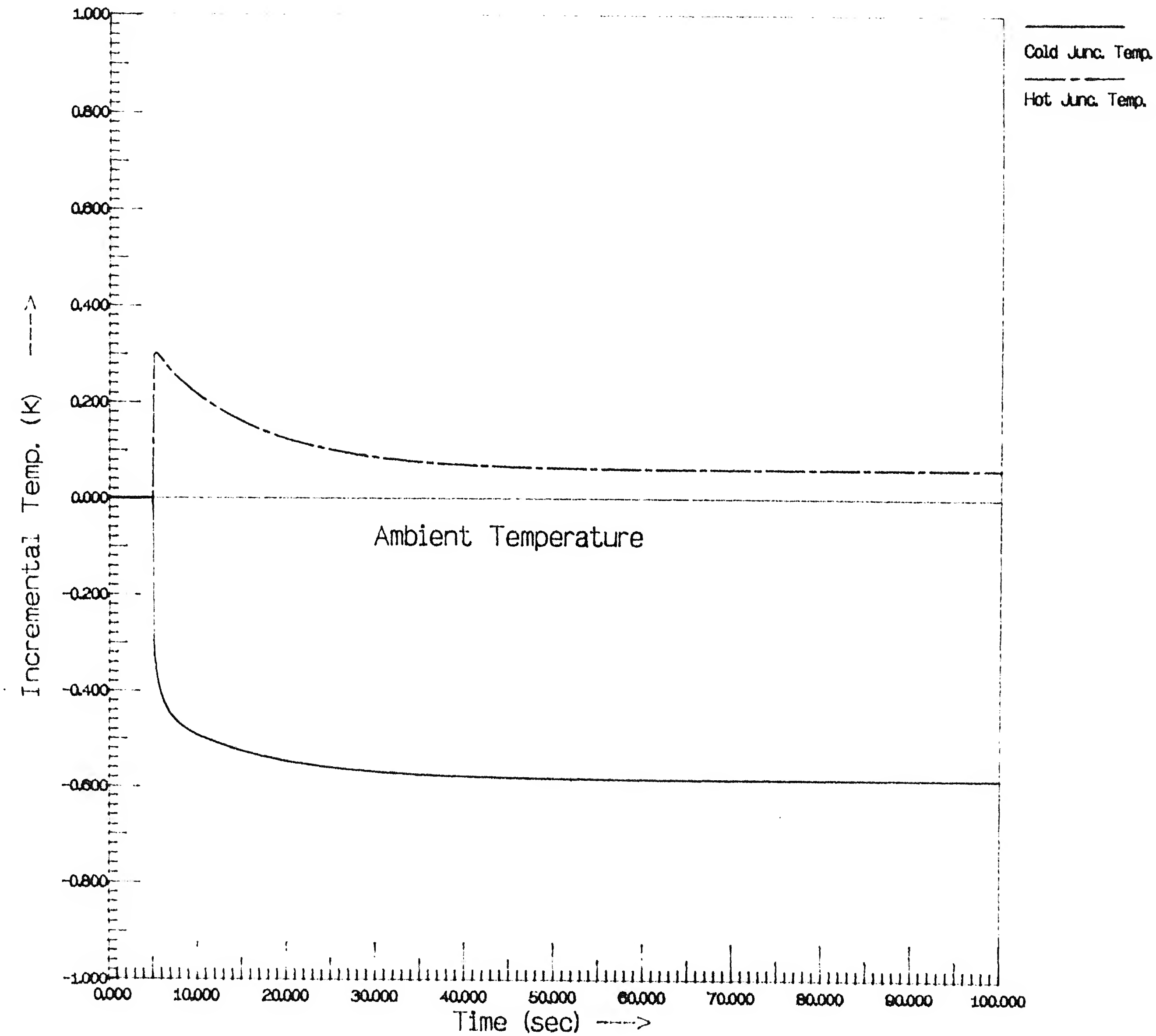


Fig. 5.1 Incremental Junction Temperatures for a Step Current of 0.5 A and Constant Power Input of 0.5 watts.

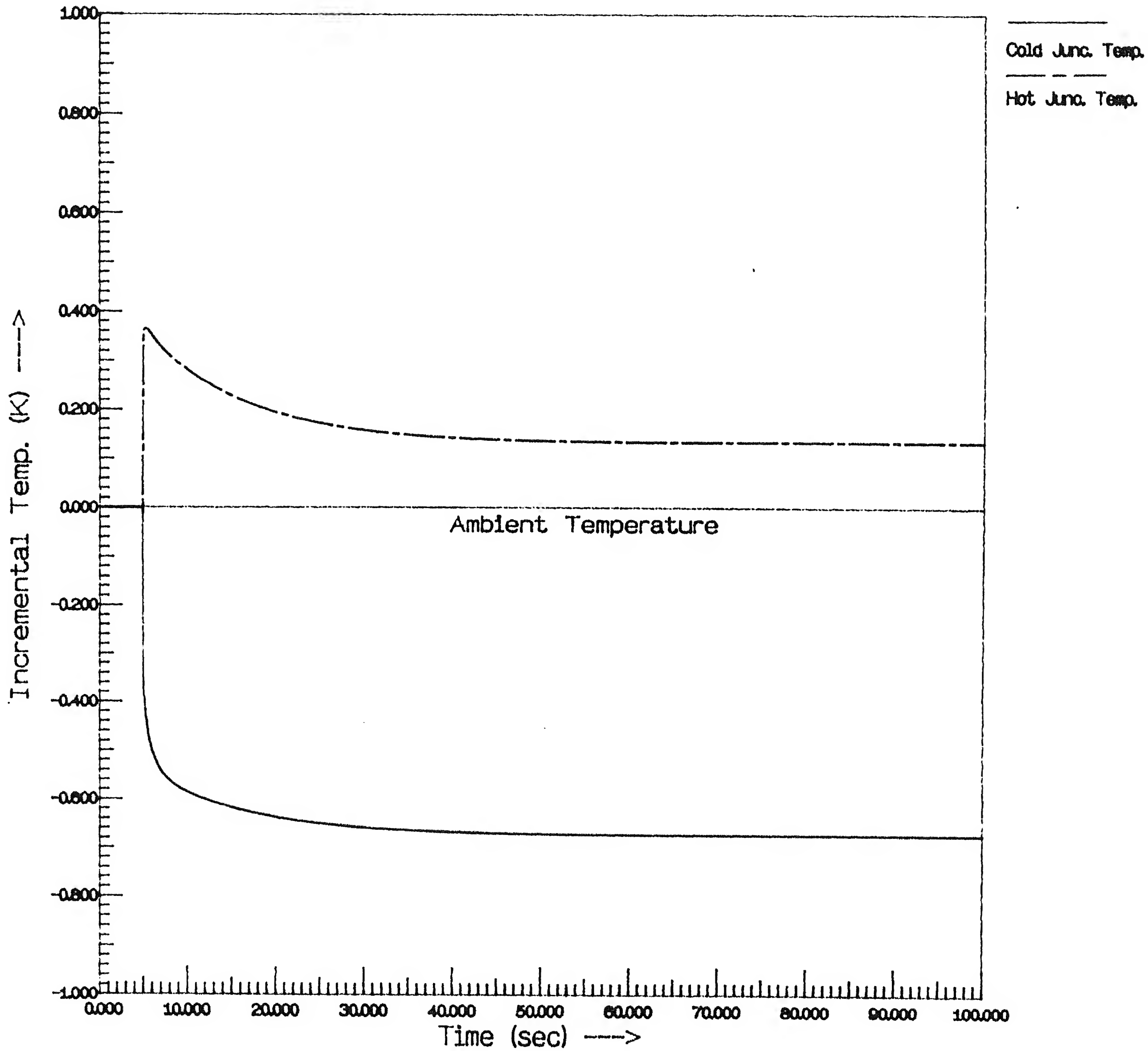


Fig. 5.2 Incremental Junction Temperatures for a Step Current of 0.75 A and Constant Power Input of 0.5 watts.

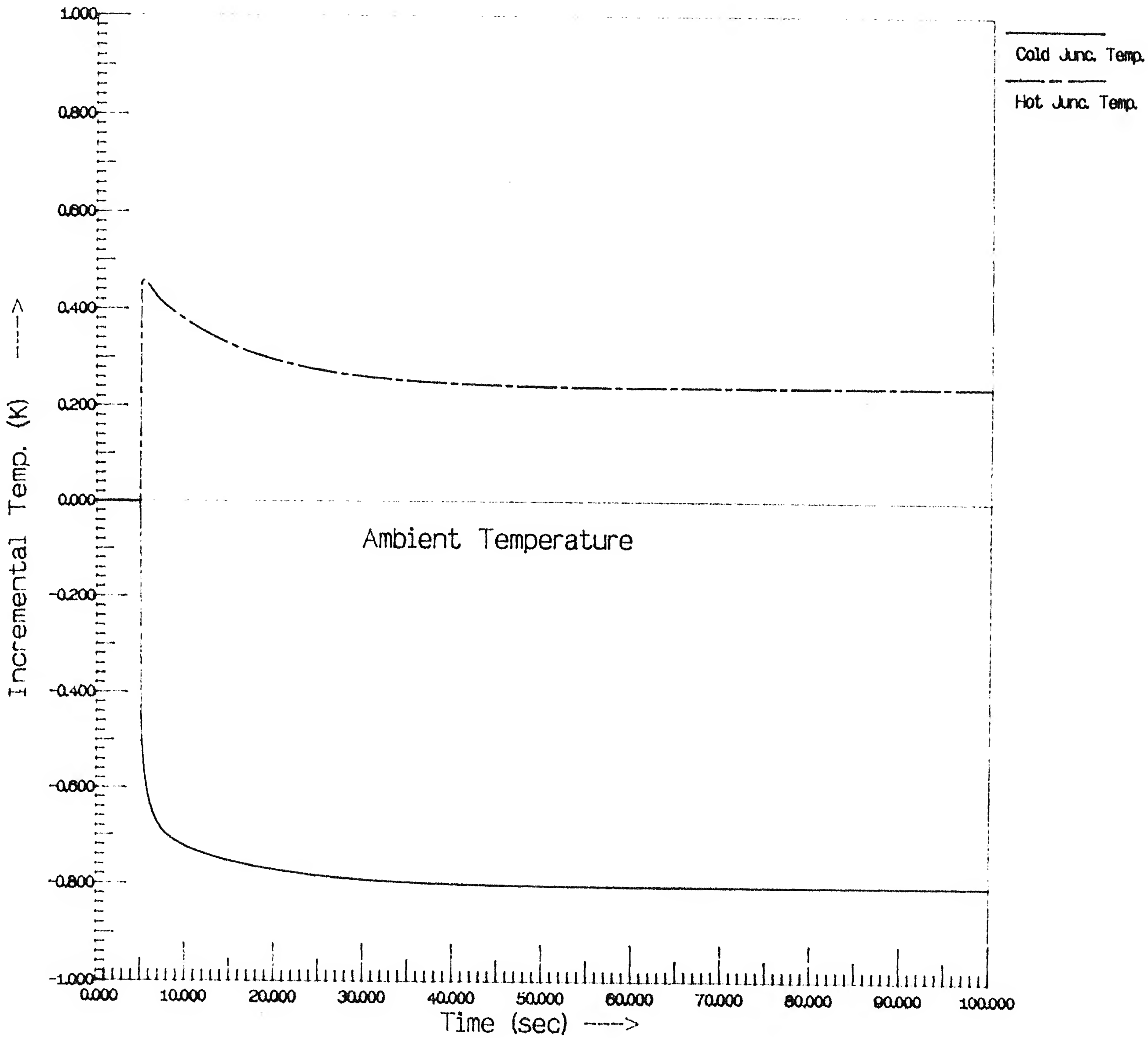


Fig. 5.3 Incremental Junction Temperatures for a Step Current of 1.0 A and Constant Power Input of 0.5 watts.

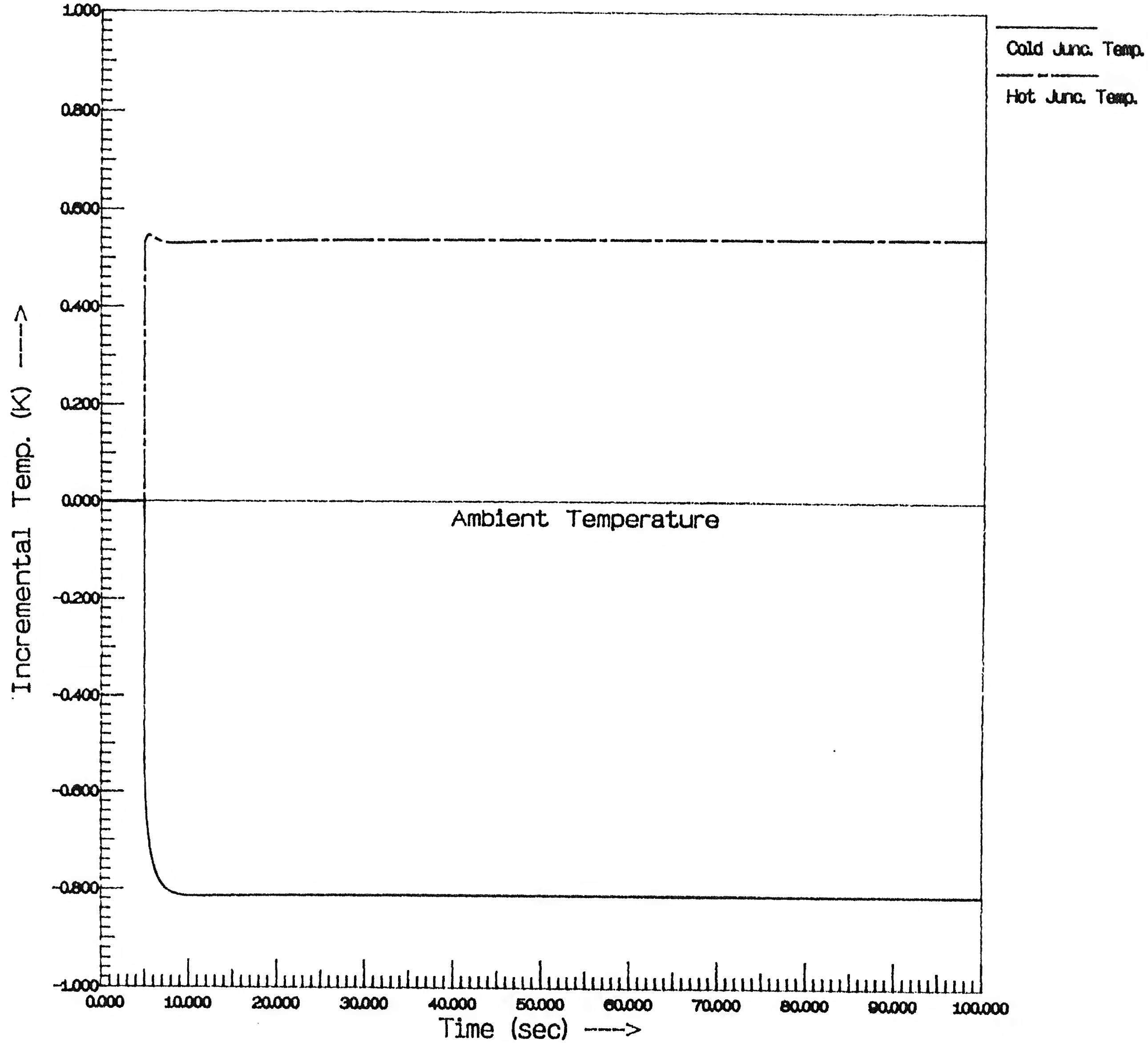


Fig. 5.4 Incremental Junction Temperatures for a Step Current of 1.5 A and Constant Power Input of 0.5 watts.

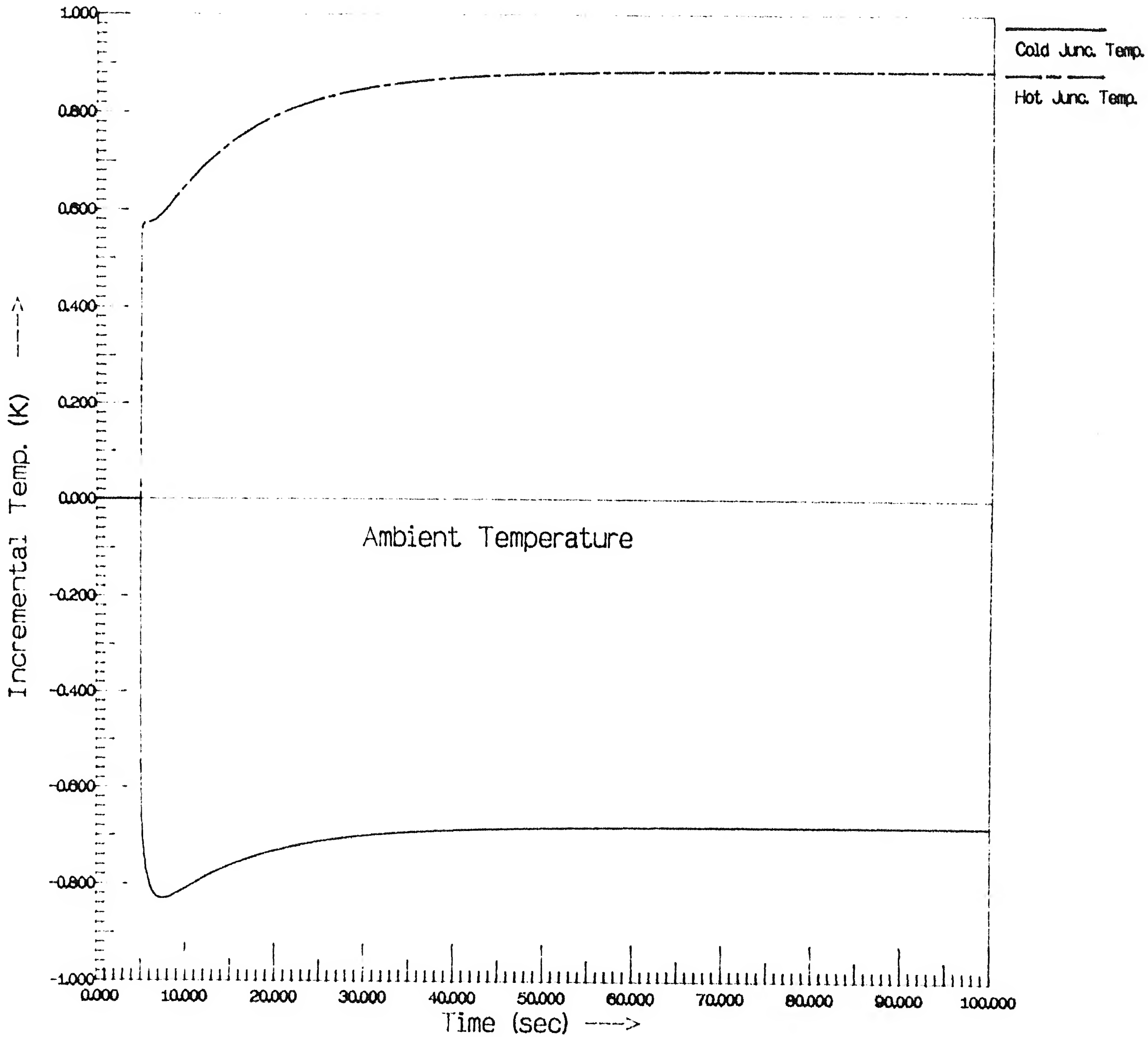


Fig. 5.5 Incremental Junction Temperatures for a Step Current of 2.0 A and Constant Power Input of 0.5 watts.

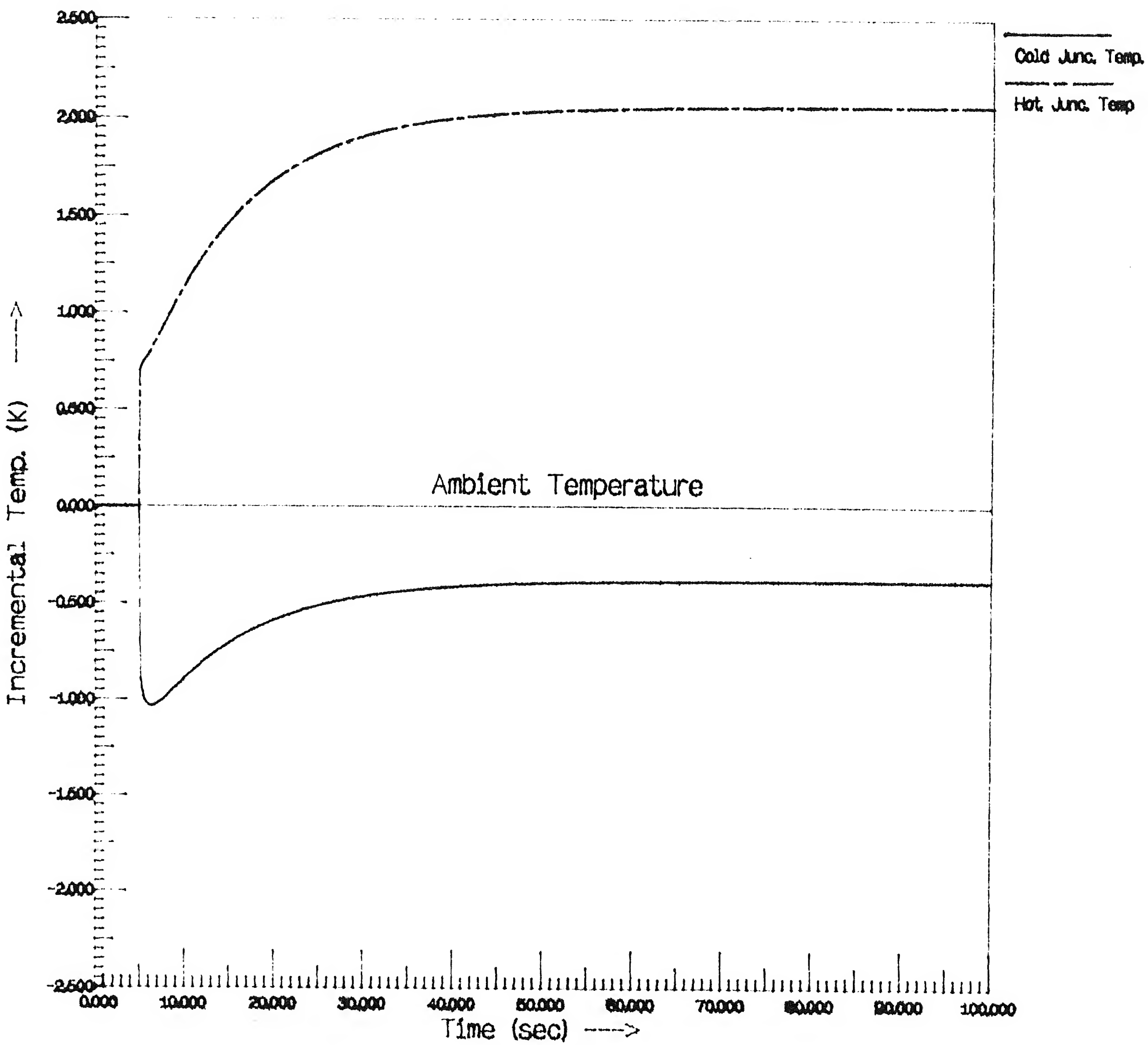


Fig. 5.6 Incremental Junction Temperatures for a Step Current of 3.0 A and Constant Power Input of 0.5 watts.

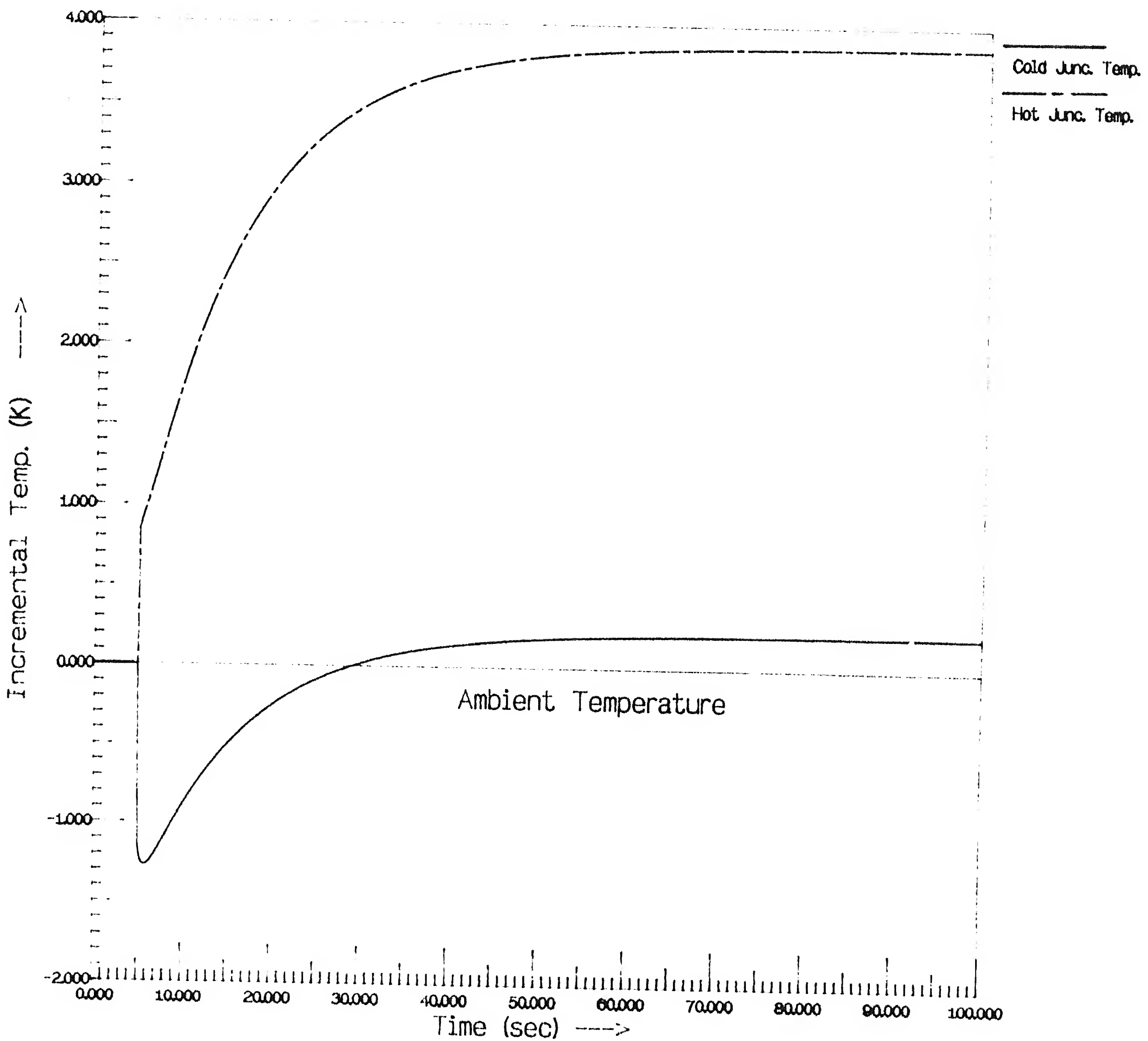


Fig. 5.7 Incremental Junction Temperatures for a Step Current of 4.0 A and Constant Power Input of 0.5 watts.

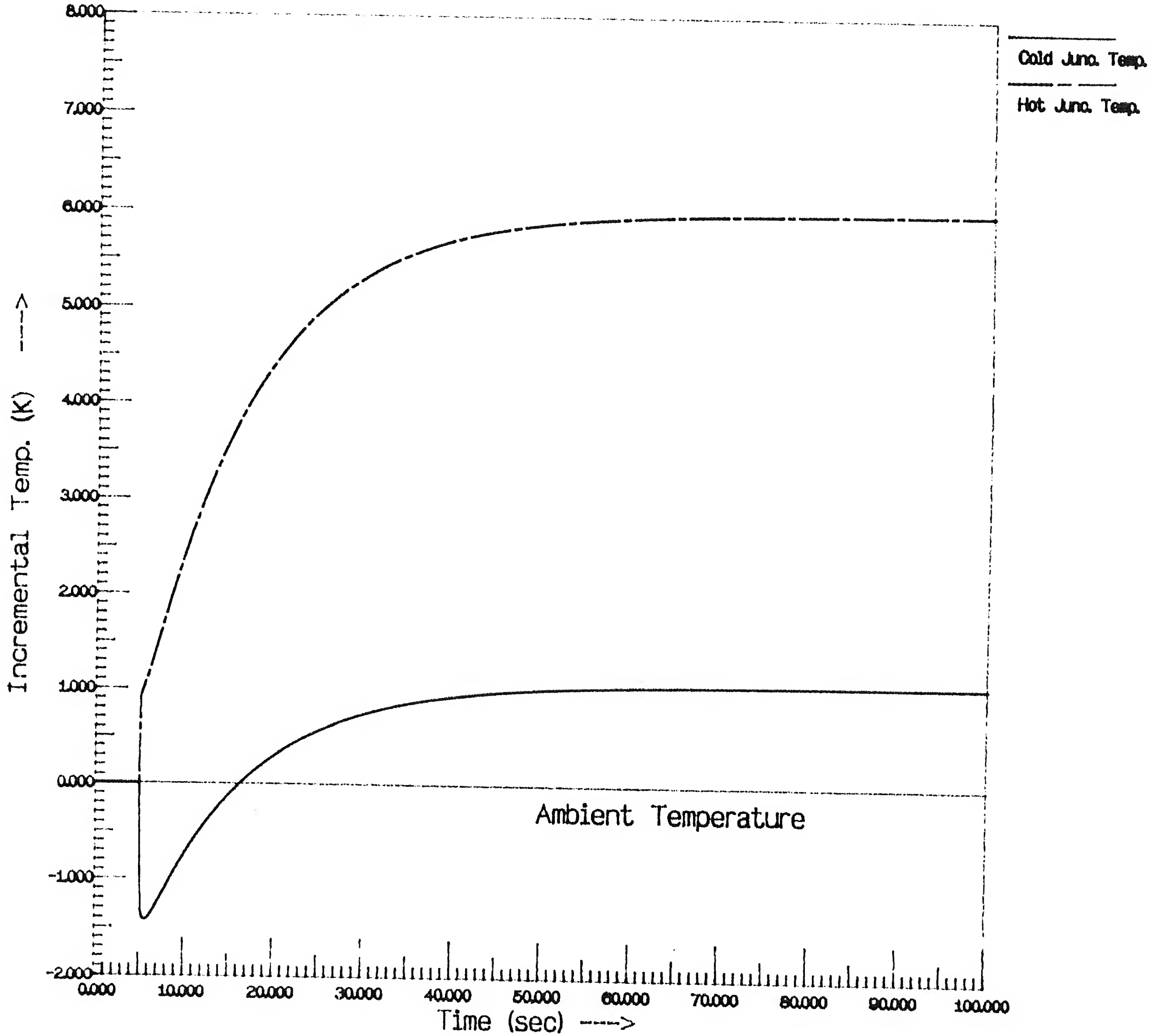


Fig. 5.8 Incremental Junction Temperatures for a Step Current of 5.0 A and Constant Power Input of 0.5 watts.

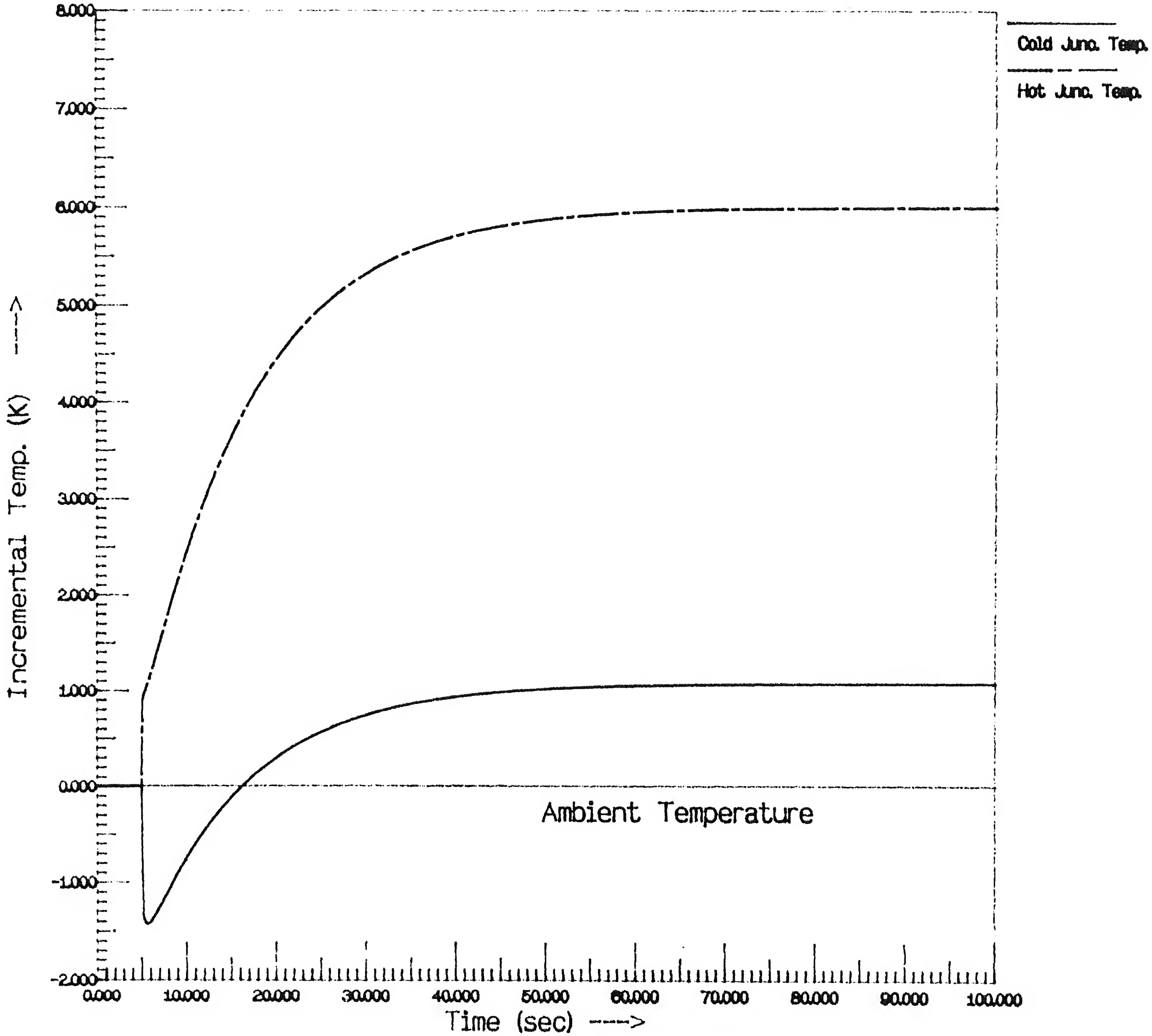


Fig. 5.8 Incremental Junction Temperatures for a Step Current of 5.0 A and Constant Power Input of 0.5 watts.

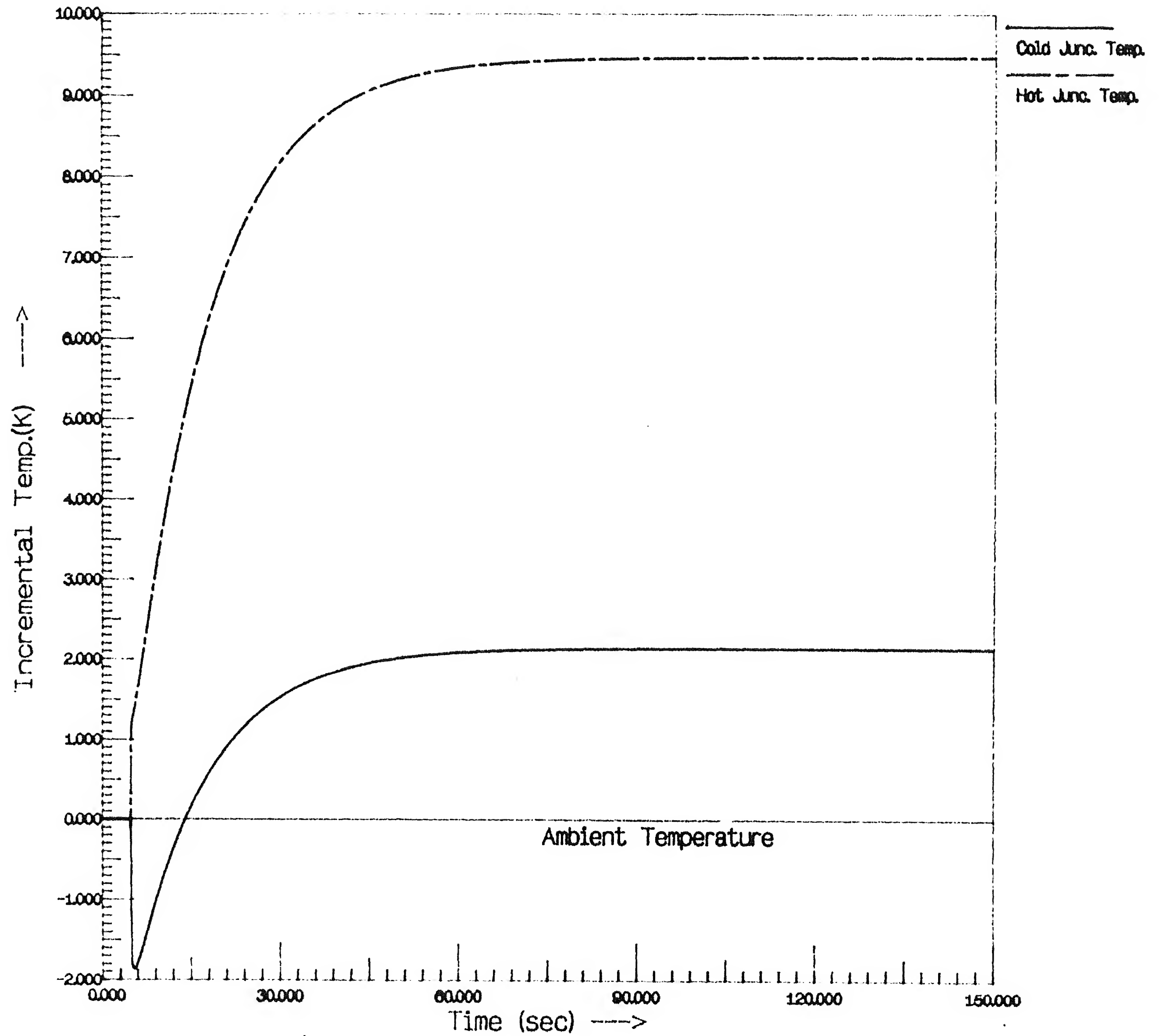


Fig. 5.9 Incremental Junction Temperatures for a Step Current of 6.0 A and Constant Power Input of 0.5 watts.

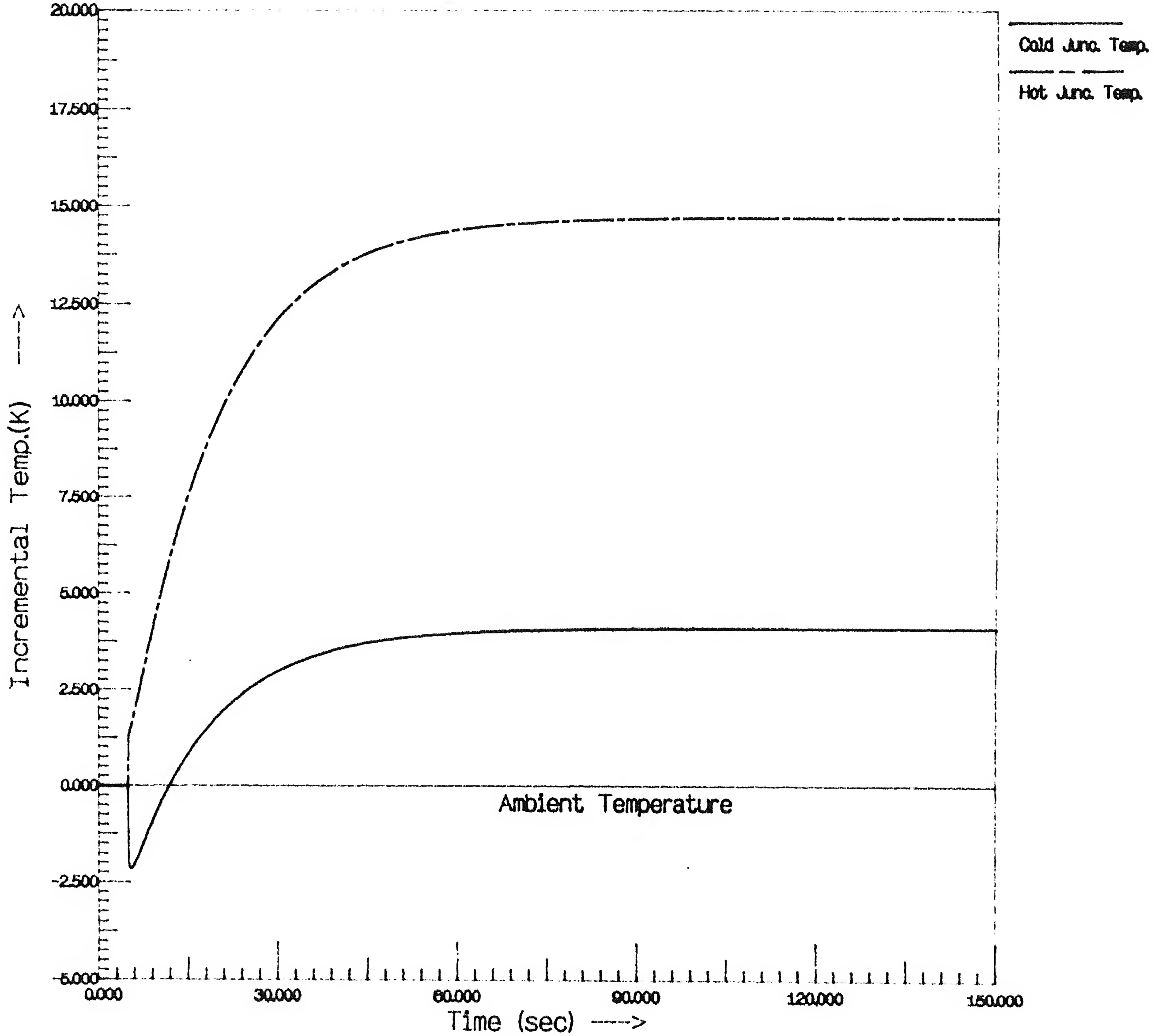


Fig. 5.10 Incremental Junction Temperatures for a Step Current of 7.0 A and Constant Power Input of 0.5 watts.

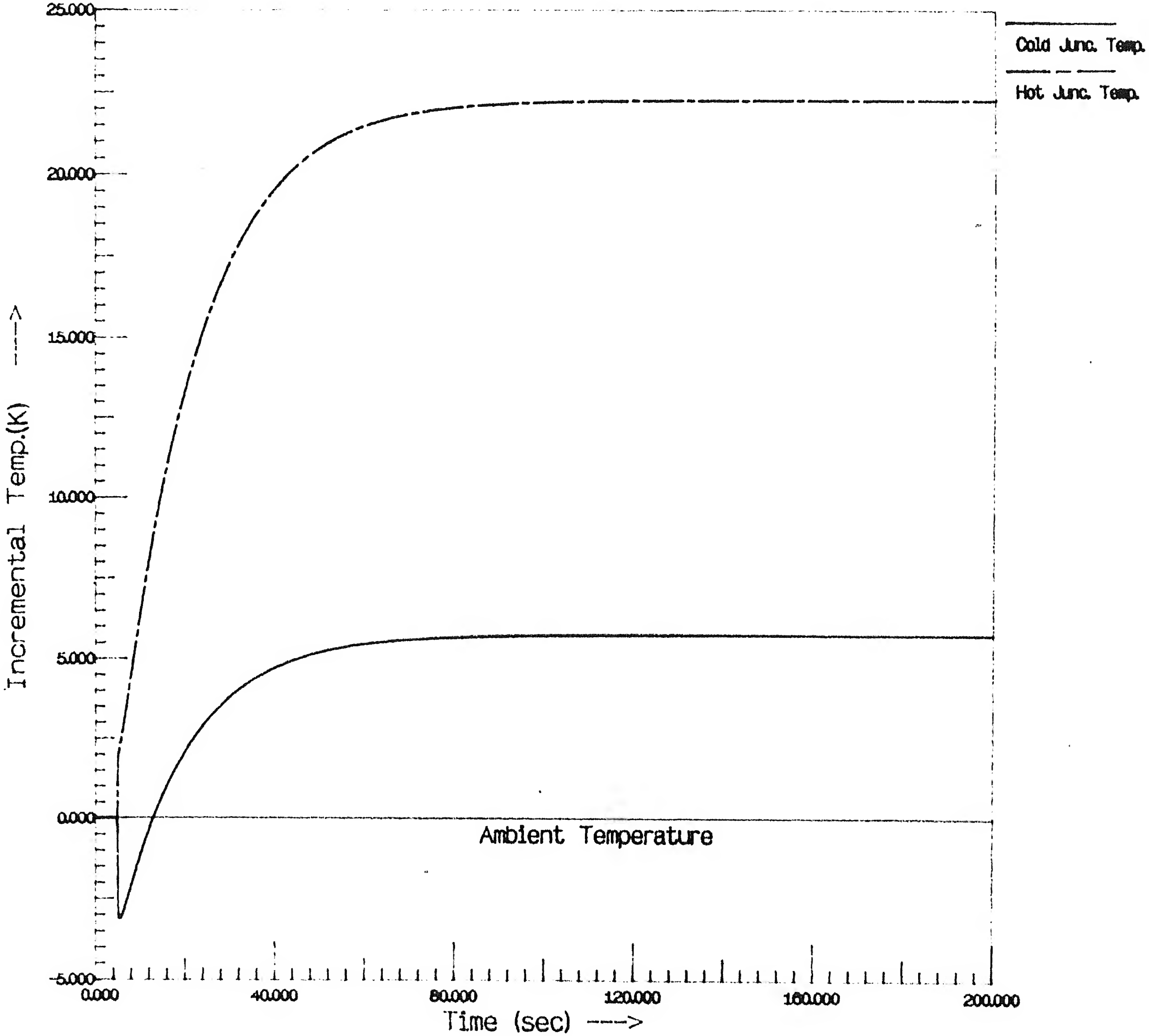


Fig. 5.11 Incremental Junction Temperatures for a Step Current of 8.0 A and Constant Power Input of 0.5 watts.

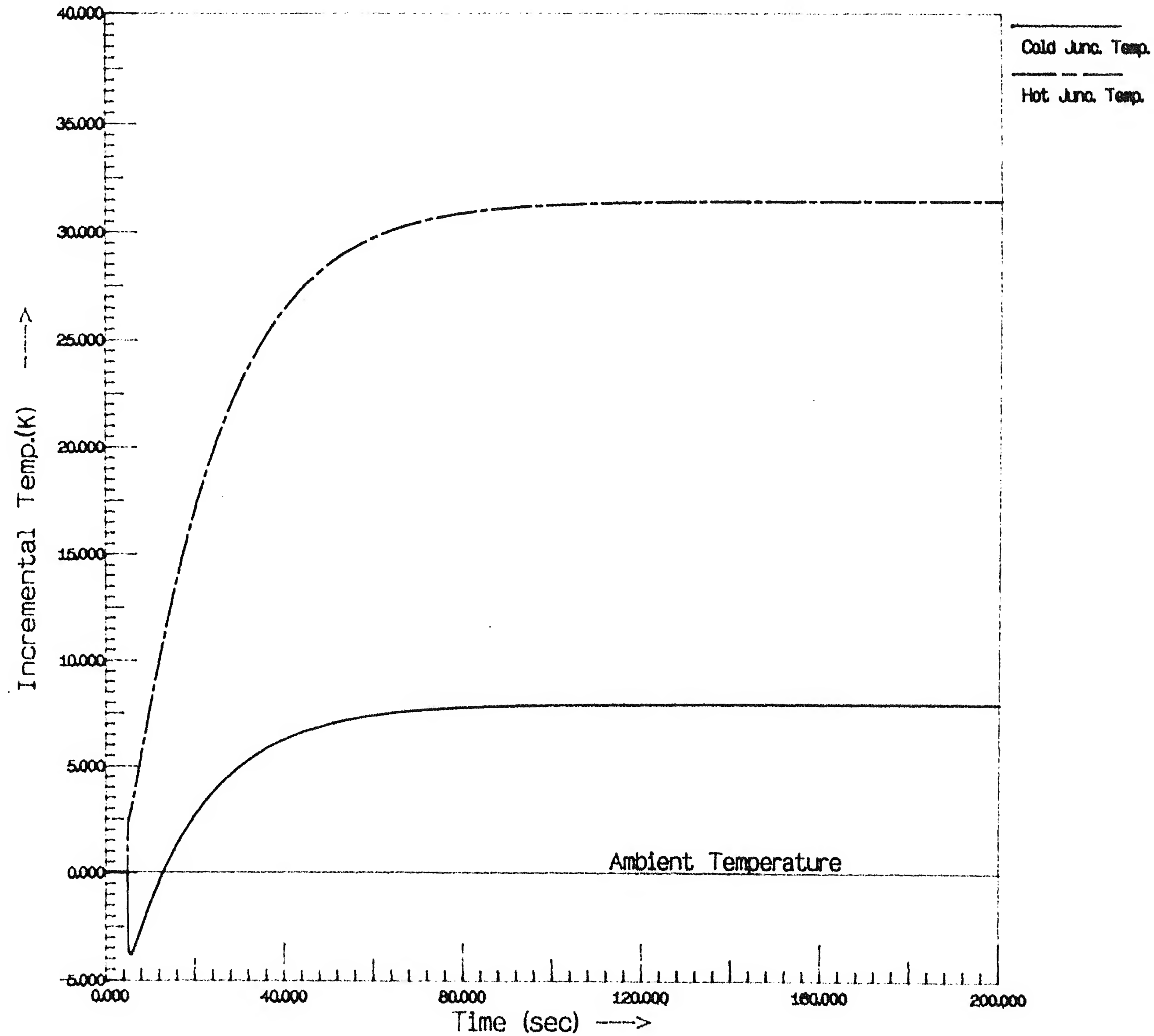


Fig. 5.12 Incremental Junction Temperatures for a Step Current of 9.0 A and Constant Power Input of 0.5 watts.

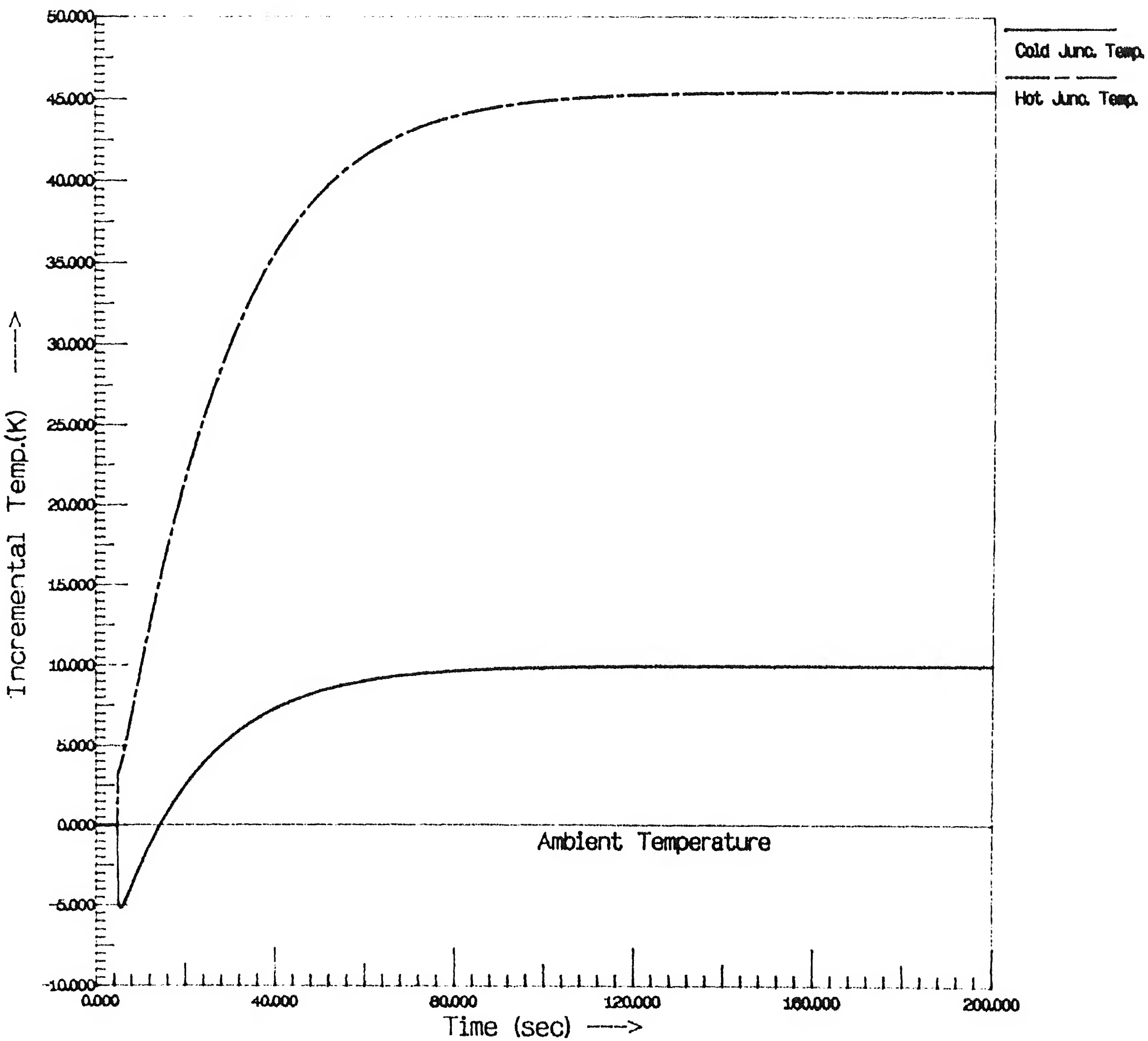


Fig. 5.13 Incremental Junction Temperatures for a Step Current of 10.0 A and Constant Power Input of 0.5 watts.

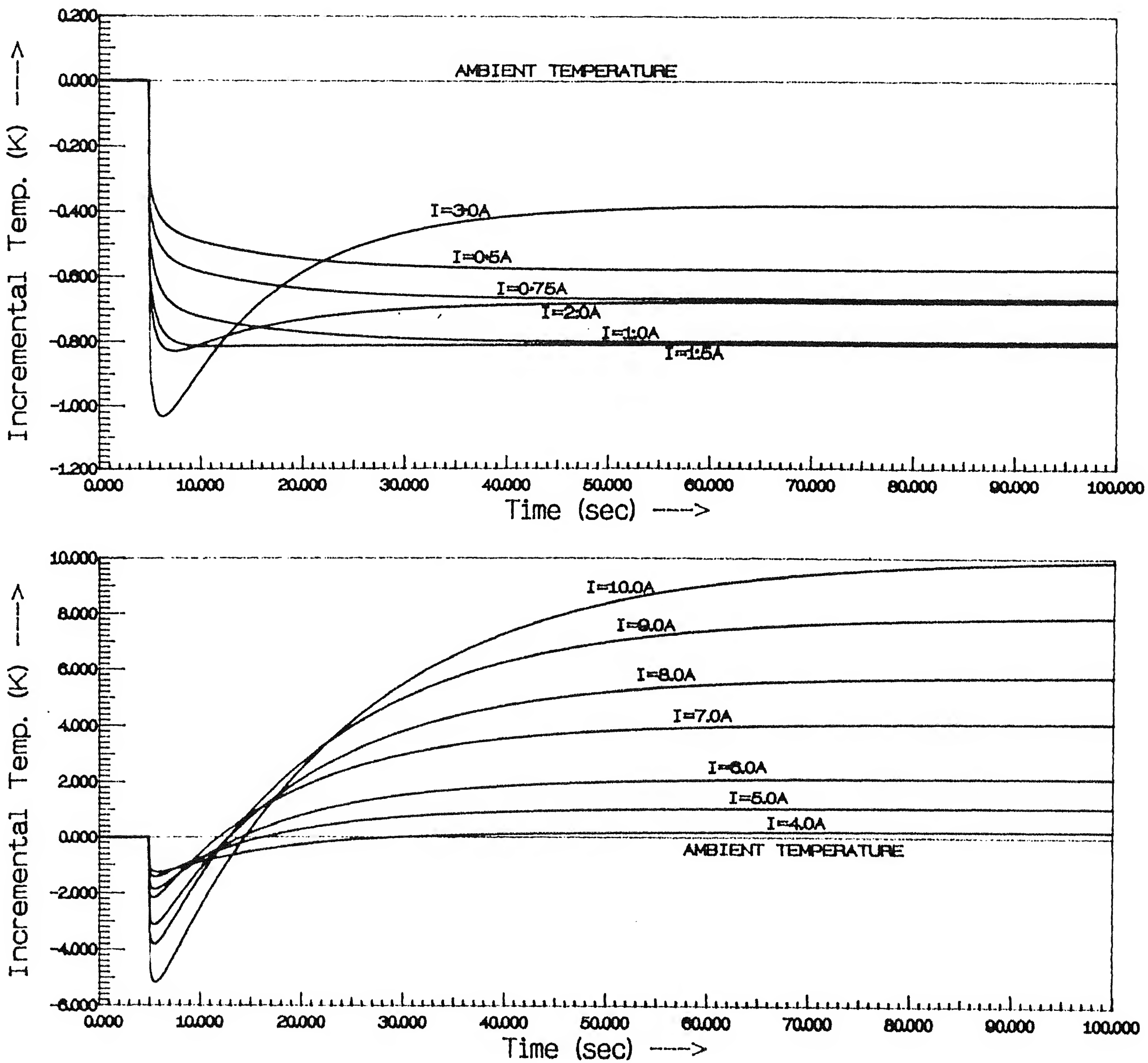


Fig. 5.14 Comparison of the Incremental Cold Junction Temperatures for Increasing Step Current Values.

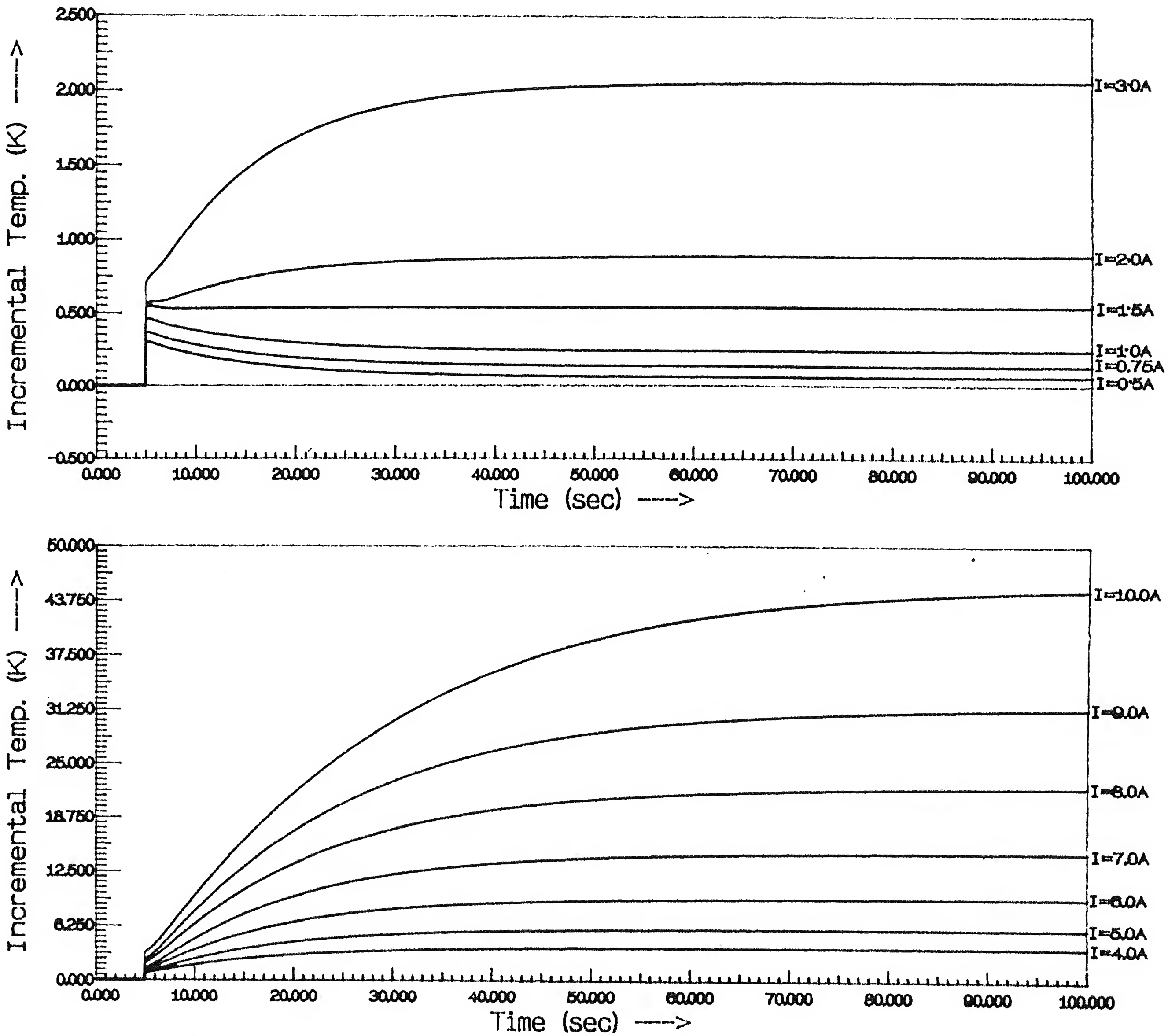
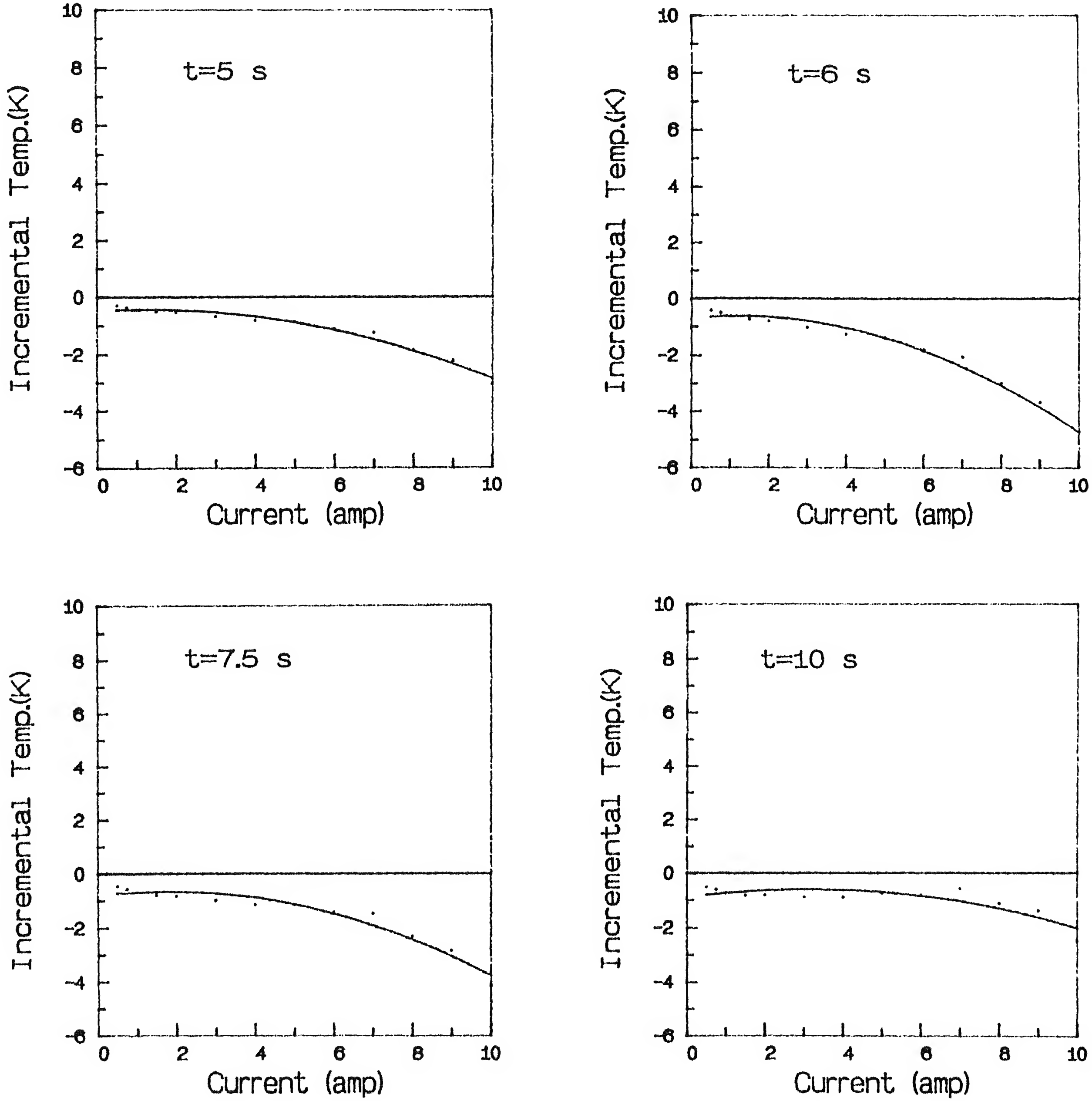
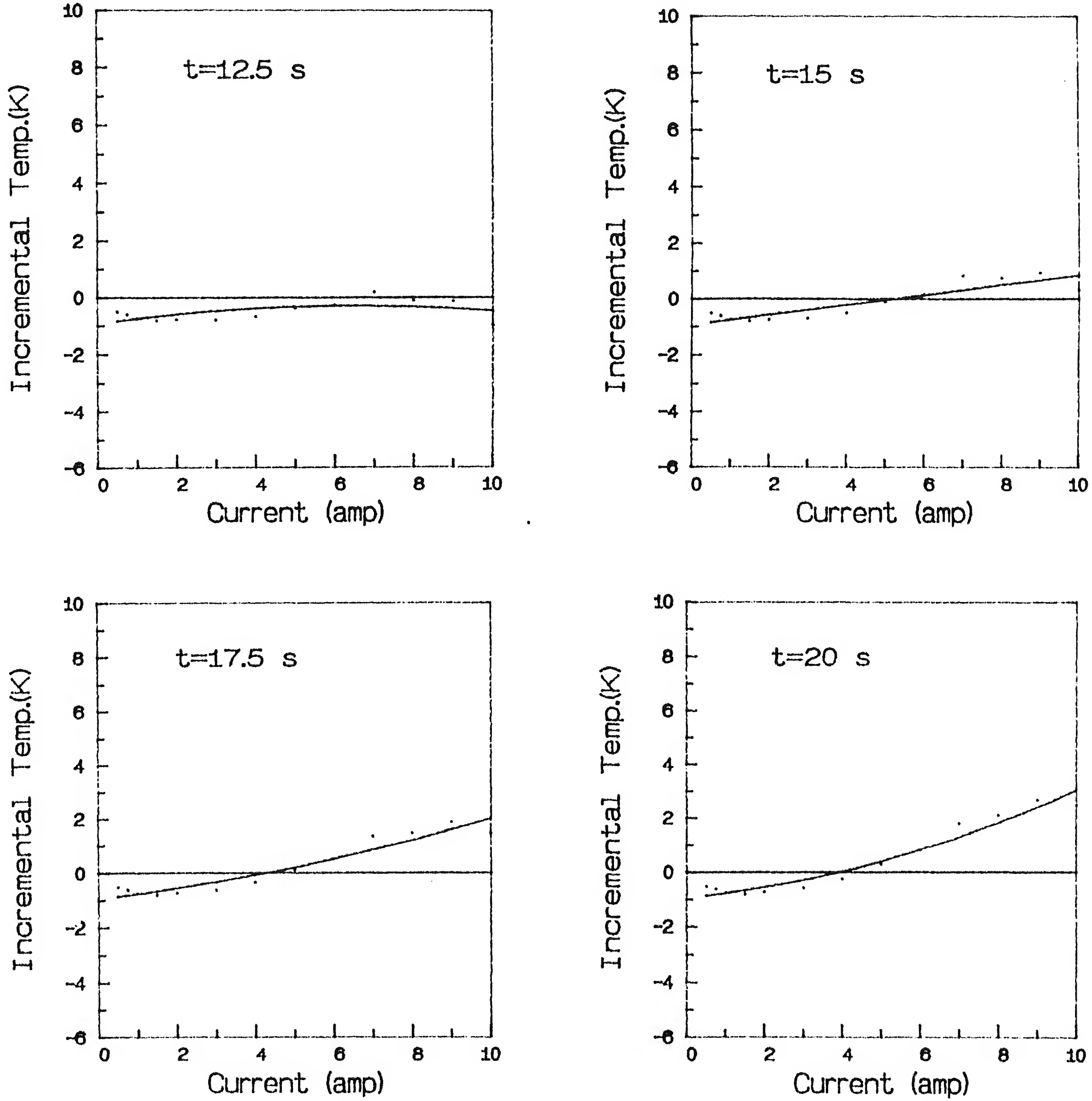
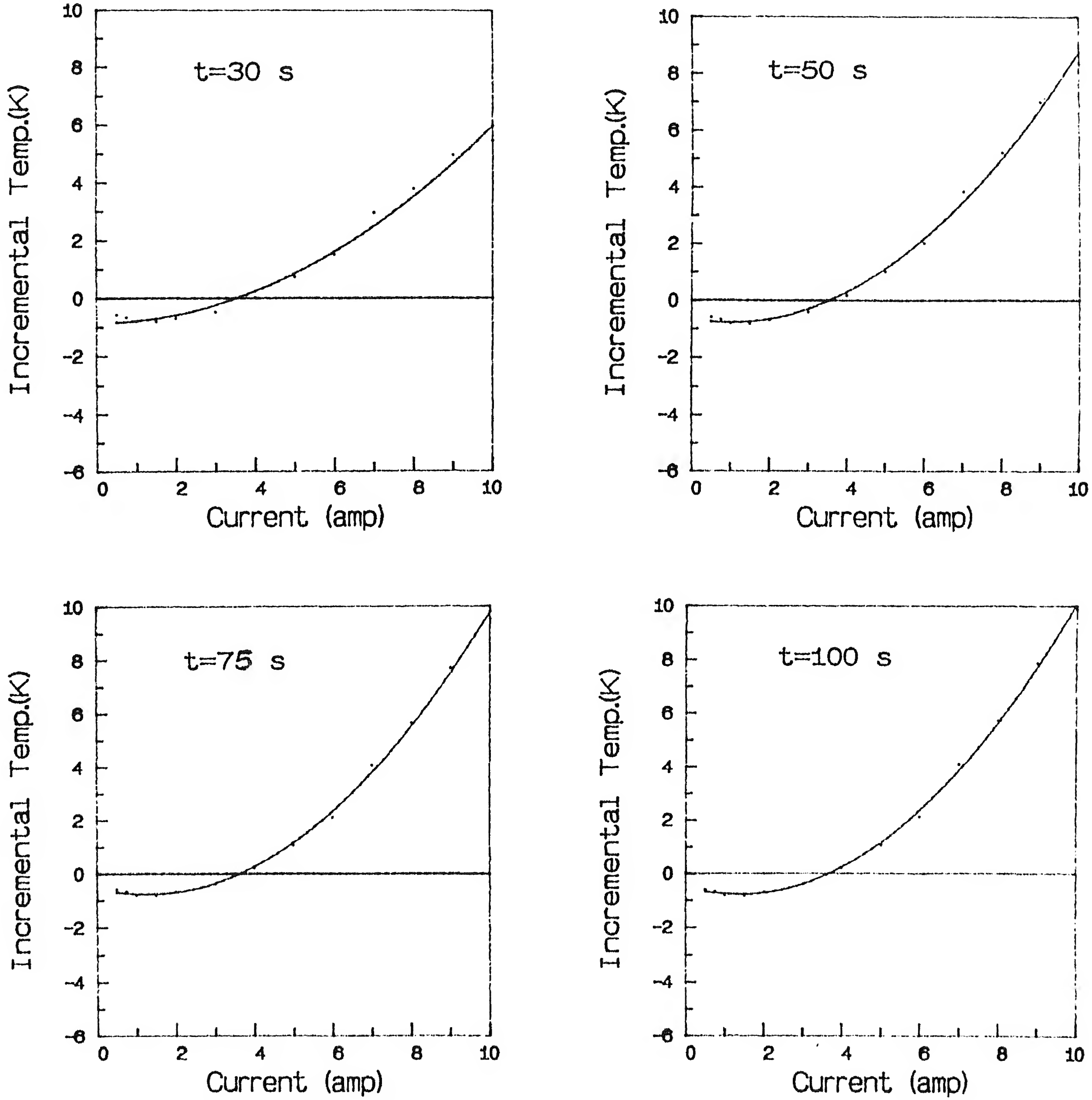


Fig. 5.15 Comparison of the Incremental Hot Junction Temperatures for Increasing Step Current Values.


t=5 s
t=6 s
t=7.5 s
t=10 s
Incremental Temp.(K)
Current (amp)



t=12.5 s
t=15 s
t=17.5 s
t=20 s
Incremental Temp.(K)
Current (amp)
10
8
6
4
2
0
-2
-4
-6


t=30 s
t=50 s
t=75 s
t=100 s
Incremental Temp.(K)
Current (amp)

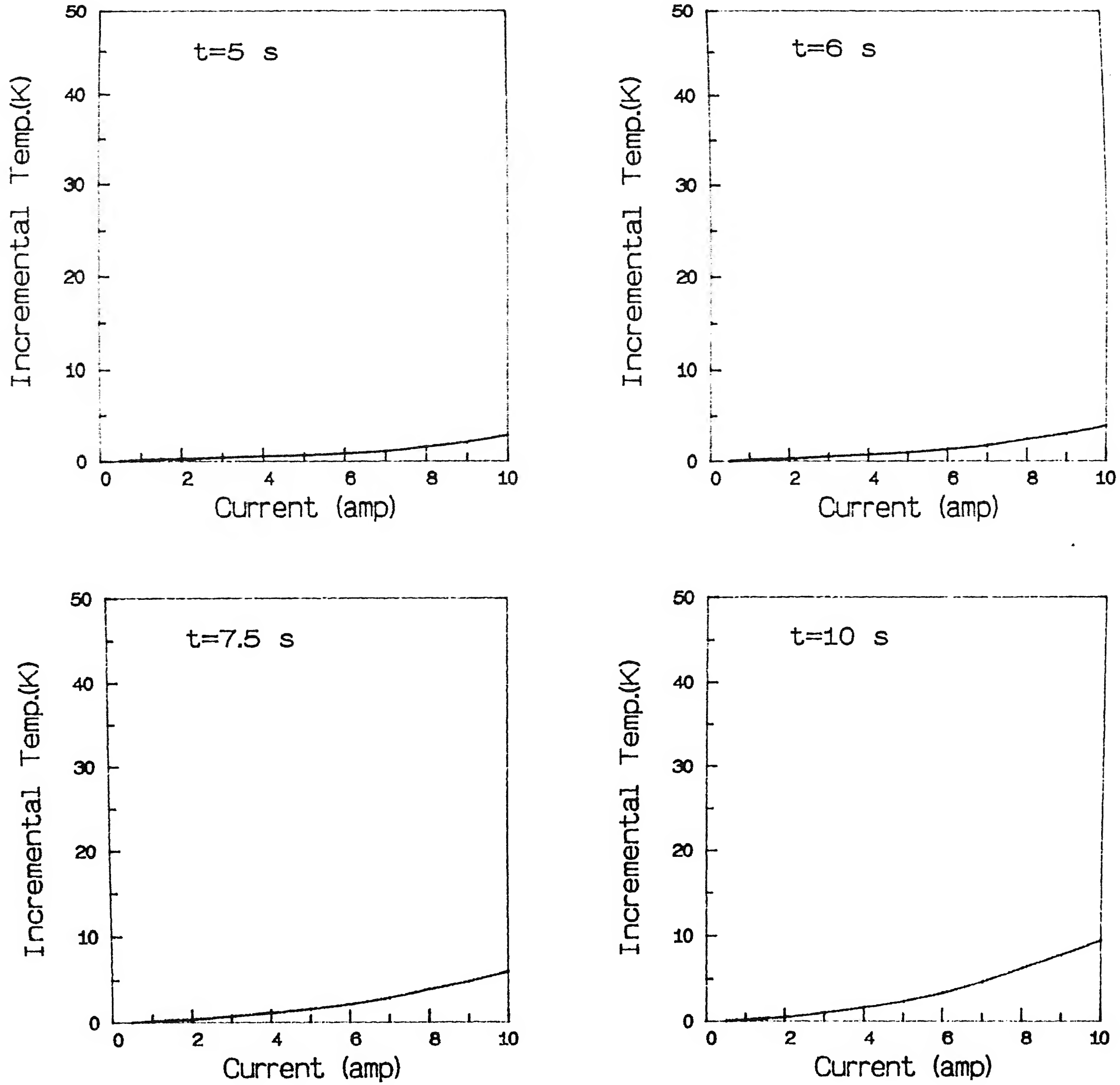


Fig. 5.17 (a) Incremental Hot Junction Temperatures for Increasing Step Current Values , at Various Instants of Time .

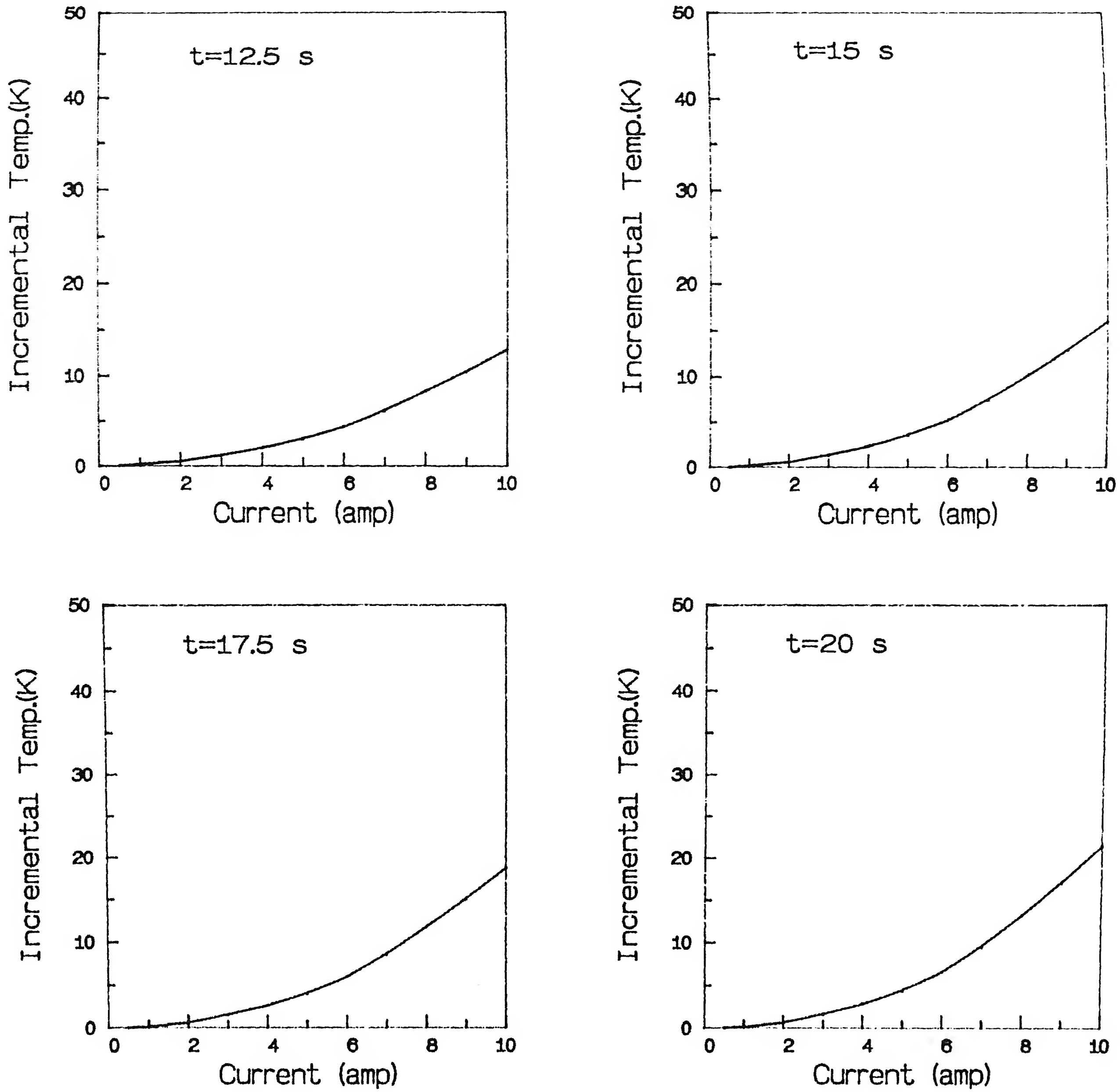


Fig. 5.17 (b) Incremental Hot Junction Temperatures for Increasing Step Current Values , at Various Instants of Time .

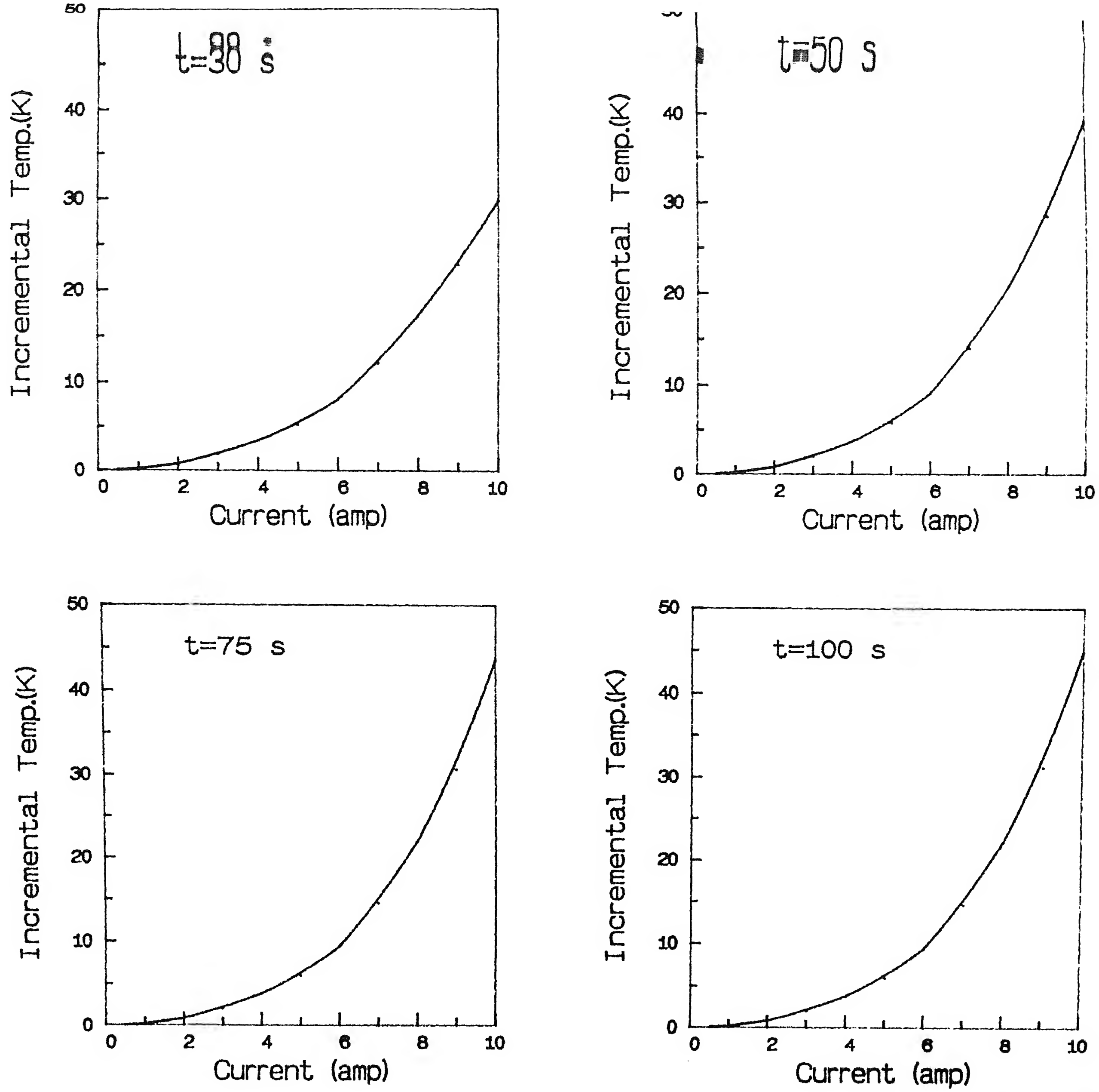


Fig. 5.17 (c) Incremental Hot Junction Temperatures for Increasing Step Current Values , at Various Instants of Time .

TABLE - 5.1

INCREMENTAL JUNCTION TEMPERATURES FOR VARIOUS STEP CURRENTS

STEP CURRENT (A)	MIN. VALUE OF THE INCREMENTAL COLD JN. TEMP. (°C)	STEADY STATE VALUE OF THE INCREMENTAL COLD JN. TEMP. (°C)	STEADY STATE VALUE OF THE INCREMENTAL HOT JN. TEMP. (°C)
0.5	- 0.583	- 0.583	0.061
0.75	- 0.672	- 0.672	0.134
1.0	- 0.804	- 0.804	0.239
1.5	- 0.813	- 0.813	0.537
2.0	- 0.830	- 0.681	0.883
3.0	- 1.033	- 0.382	2.058
4.0	- 1.271	0.217	3.869
5.0	- 1.427	1.075	5.982
6.0	- 1.864	2.140	9.466
7.0	- 2.168	4.100	14.687
8.0	- 3.121	5.756	22.256
9.0	- 3.820	7.921	31.407
10.0	- 5.174	9.969	45.419

CASE - i(b) : Sinusoidal change in the current with the power input at the cold junction kept constant at 0.5 W.

The current is taken to be of the form :

$$I = I_{01} + I_0 . \sin(\omega . t)$$

where I_{01} is the fixed component of the current while I_0 is the amplitude of the variable part. Two values of the current are considered, namely :

$I_{01} = I_0 = 0.5$ A , and

$I_{01} = I_0 = 5.0$ A .

Three values of the frequency, ω, are taken for each case - 0.1 Hz., 1.0 Hz. and 10.0 Hz. The incremental junction temperatures have been plotted against time in Figs.5.18 through 5.23. The current is also plotted above the temperatures in each case.

The figures illustrate the phase behaviour of the device under a sinusoidal current input. The hot and the cold junction temperature are always in phase with one another and have the same frequency as the current. However they have a definite phase difference with the current, which increases as the frequency of the input current increases. For smaller current values the phase difference increases at a slower rate with increasing frequencies. The phase difference for the two current values and three frequencies are given in Table 5.2.

The phase difference occurs because the temperature of the thermocouples is not a linear function of the current, since Joulean heat generation is a function of the square of the current. This leads to a phase difference between the current and the temperature. The linear components (Peltier effect and heat conduction) have a proportionately larger effect at low current values, as compared to the Joulean heat and so the phase differences too increase at a slower rate for smaller currents.

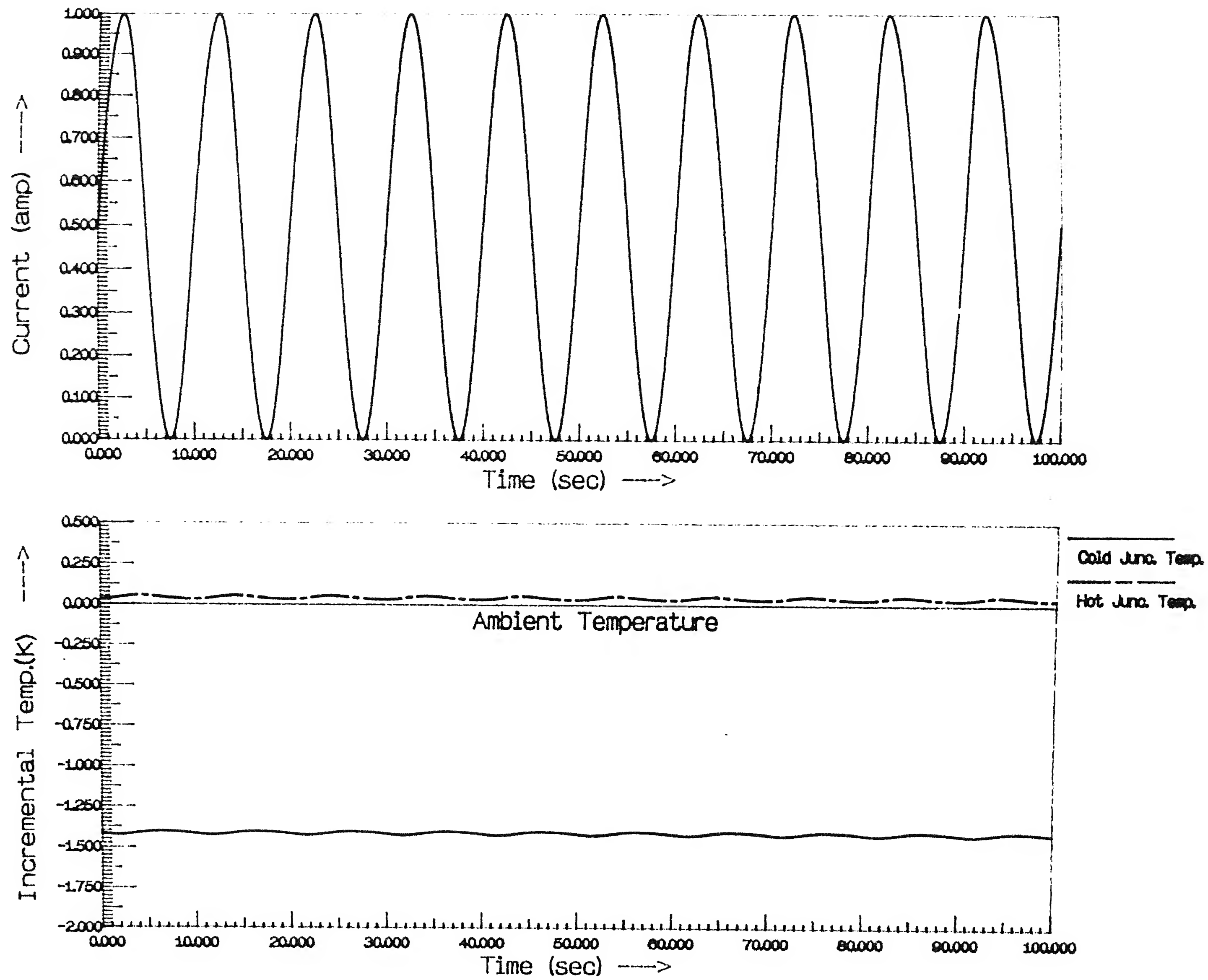


Fig. 5.18 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 0.5 A and Frequency 0·1 Hz. with a Constant Power Input of 0.5 watts.

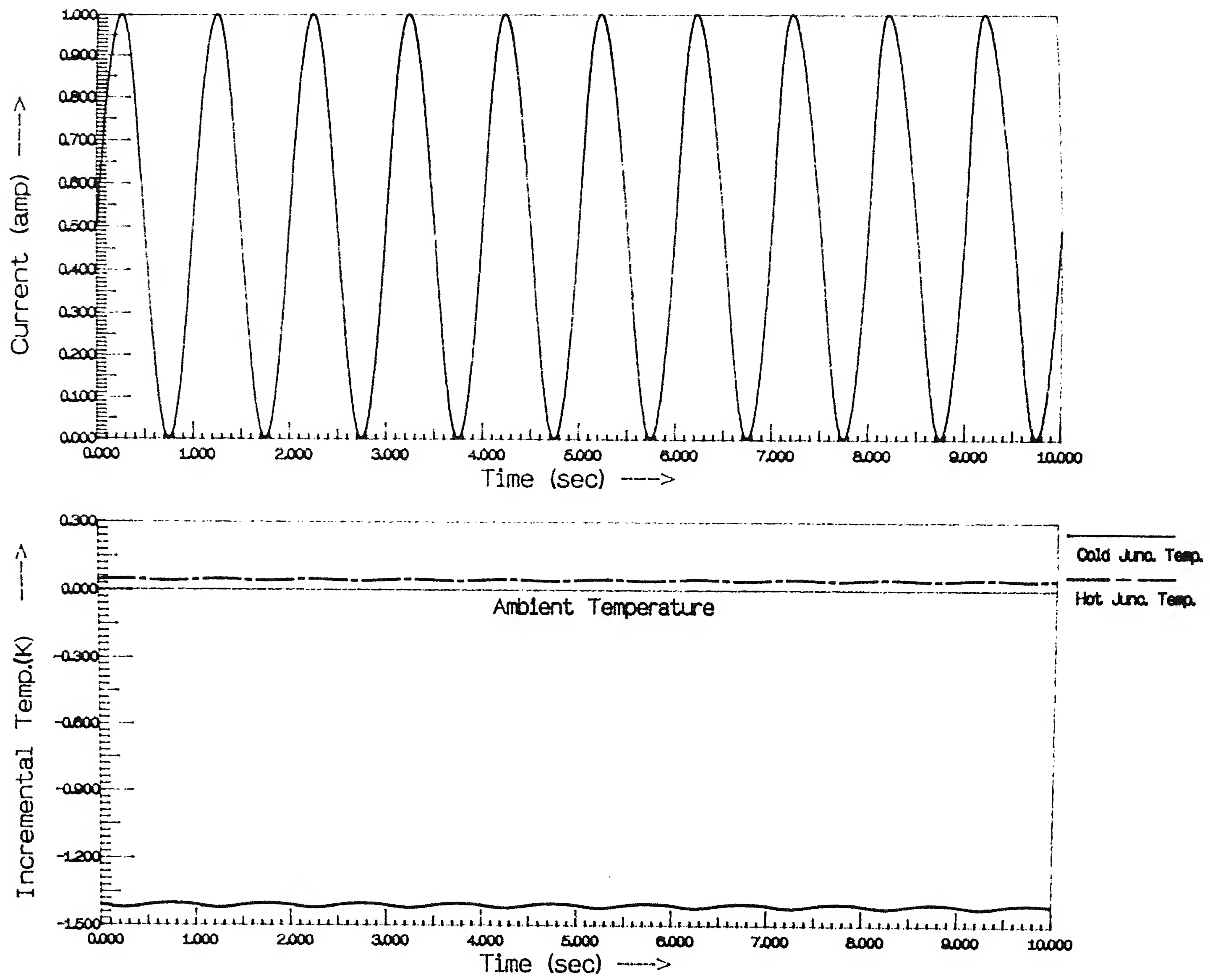


Fig. 5.19 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 0.5 A and Frequency 1·0 Hz. with a Constant Power Input of 0.5 watts.

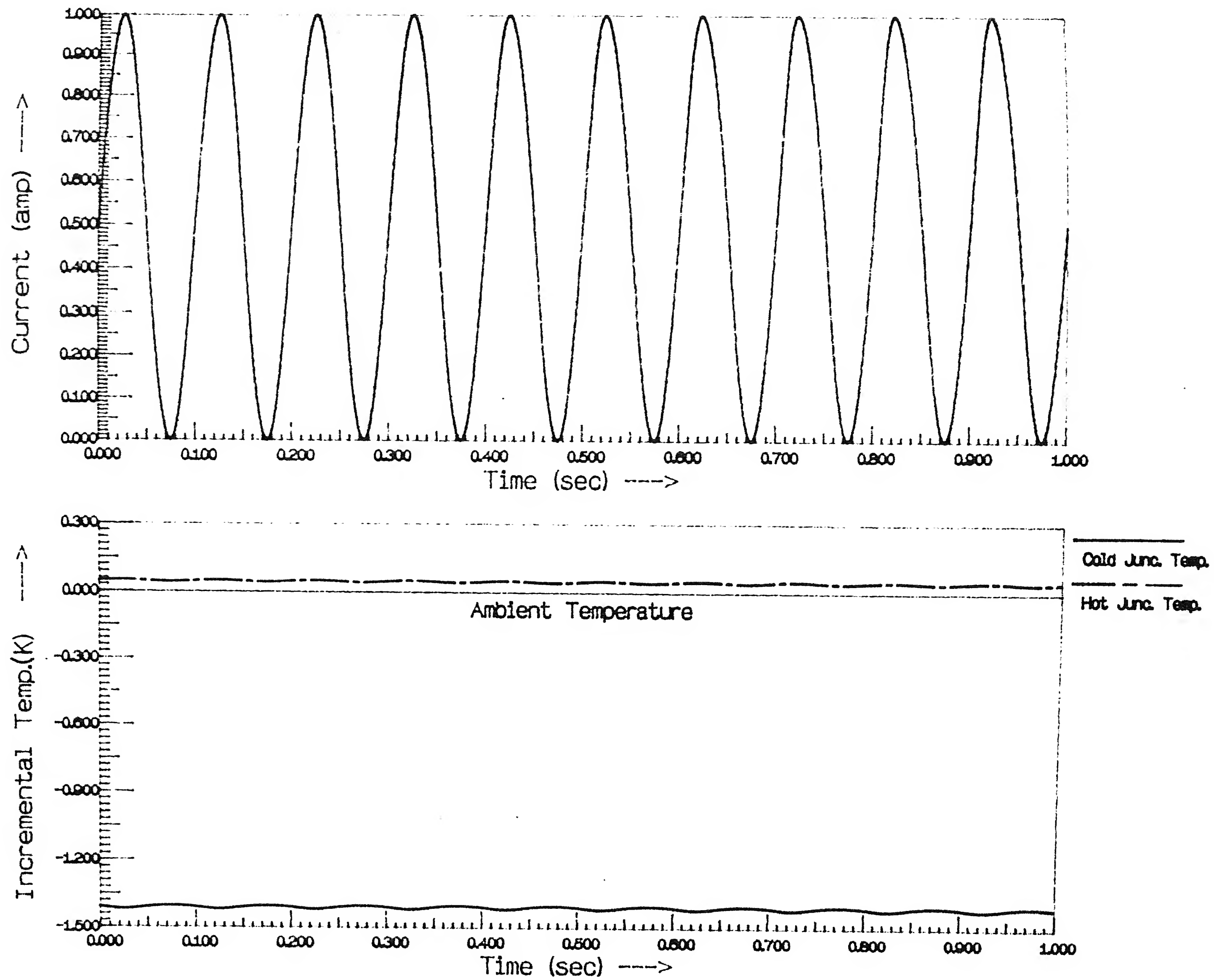


Fig. 5.20 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 0.5 A and Frequency 10 Hz. with a Constant Power Input of 0.5 watts.

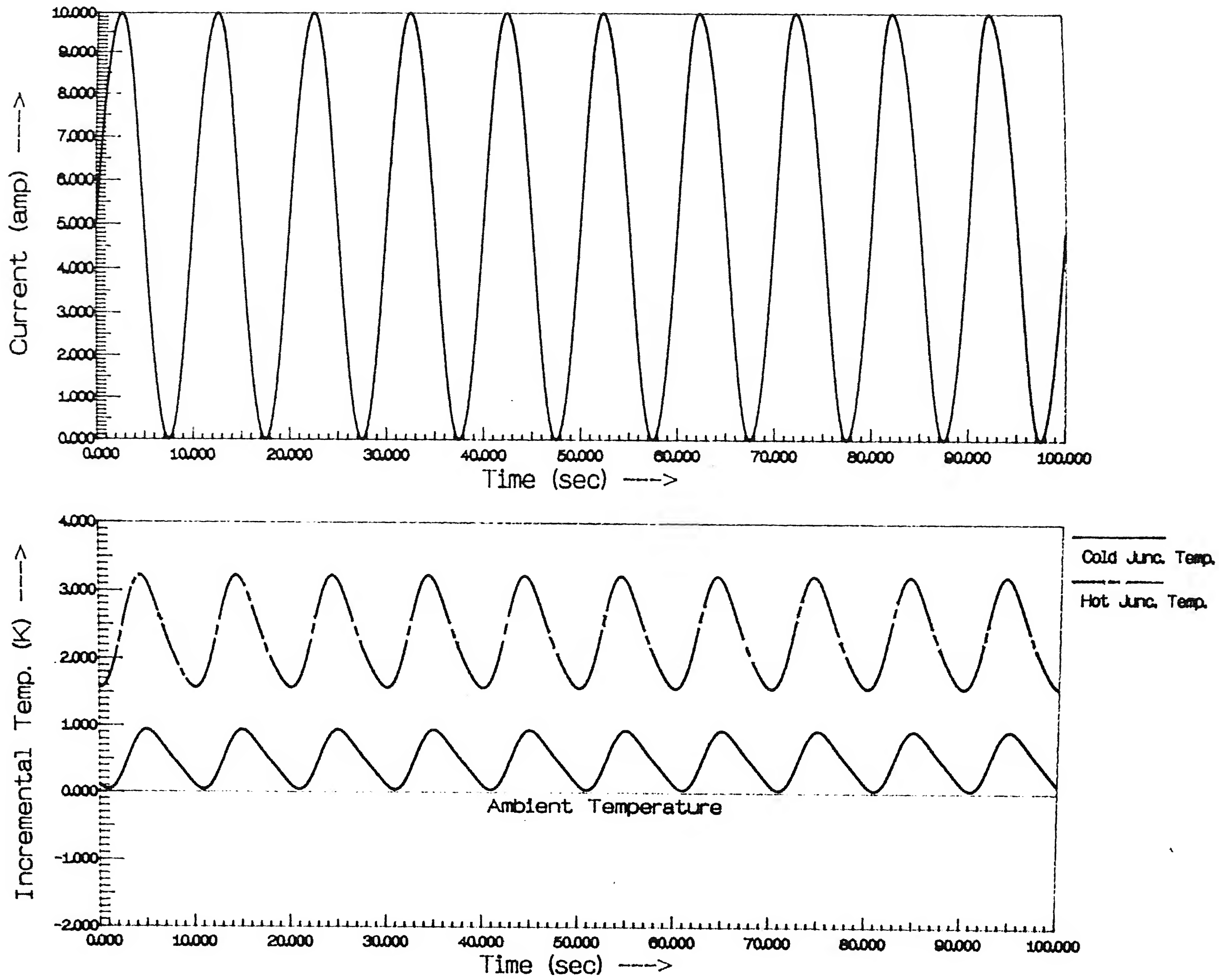


Fig. 5.21 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 5.0 A and Frequency 0.1 Hz. with a Constant Power Input of 0.5 watts.

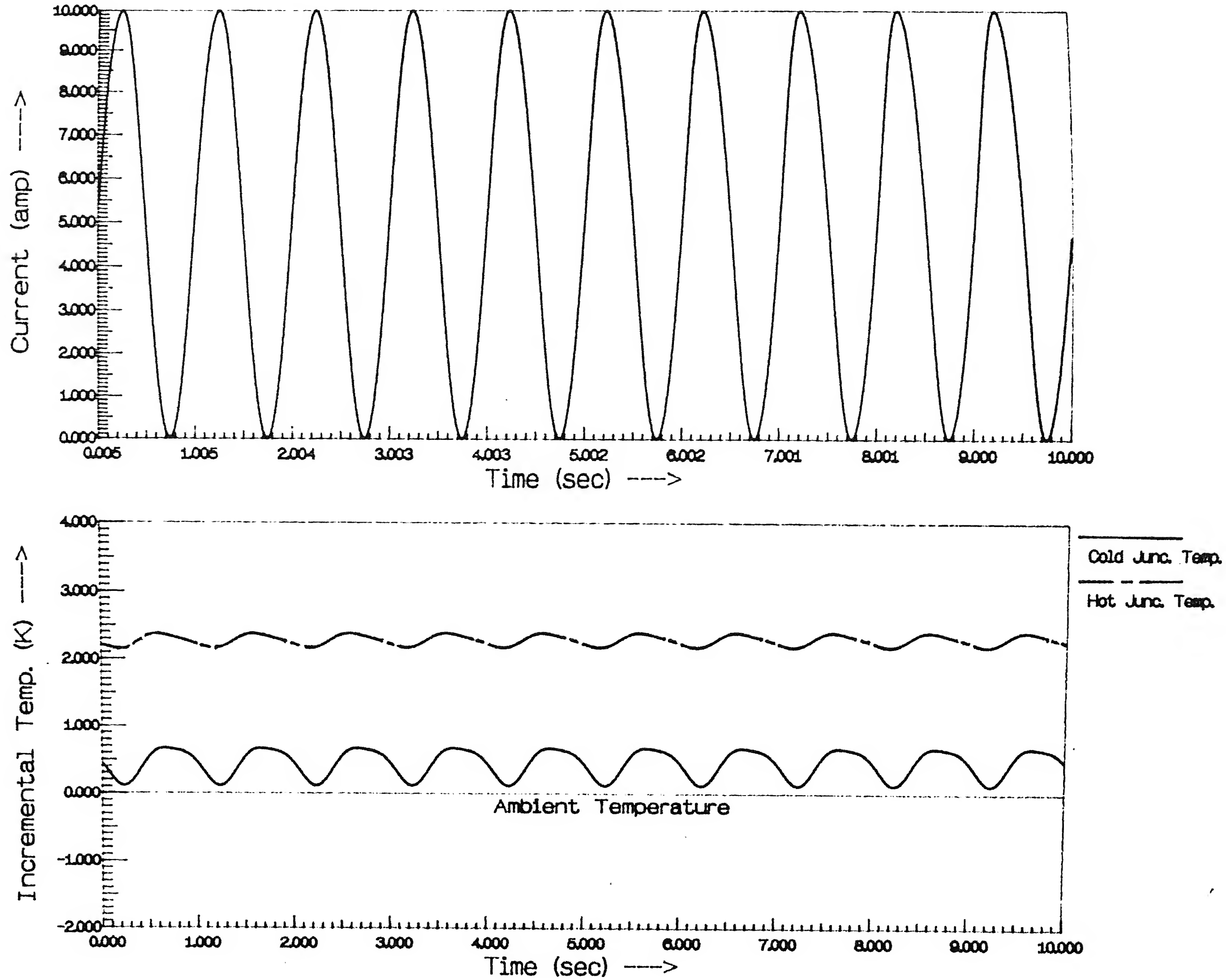


Fig. 5.22 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 5.0 A and Frequency 1.0 Hz. with a Constant Power Input of 0.5 watts.

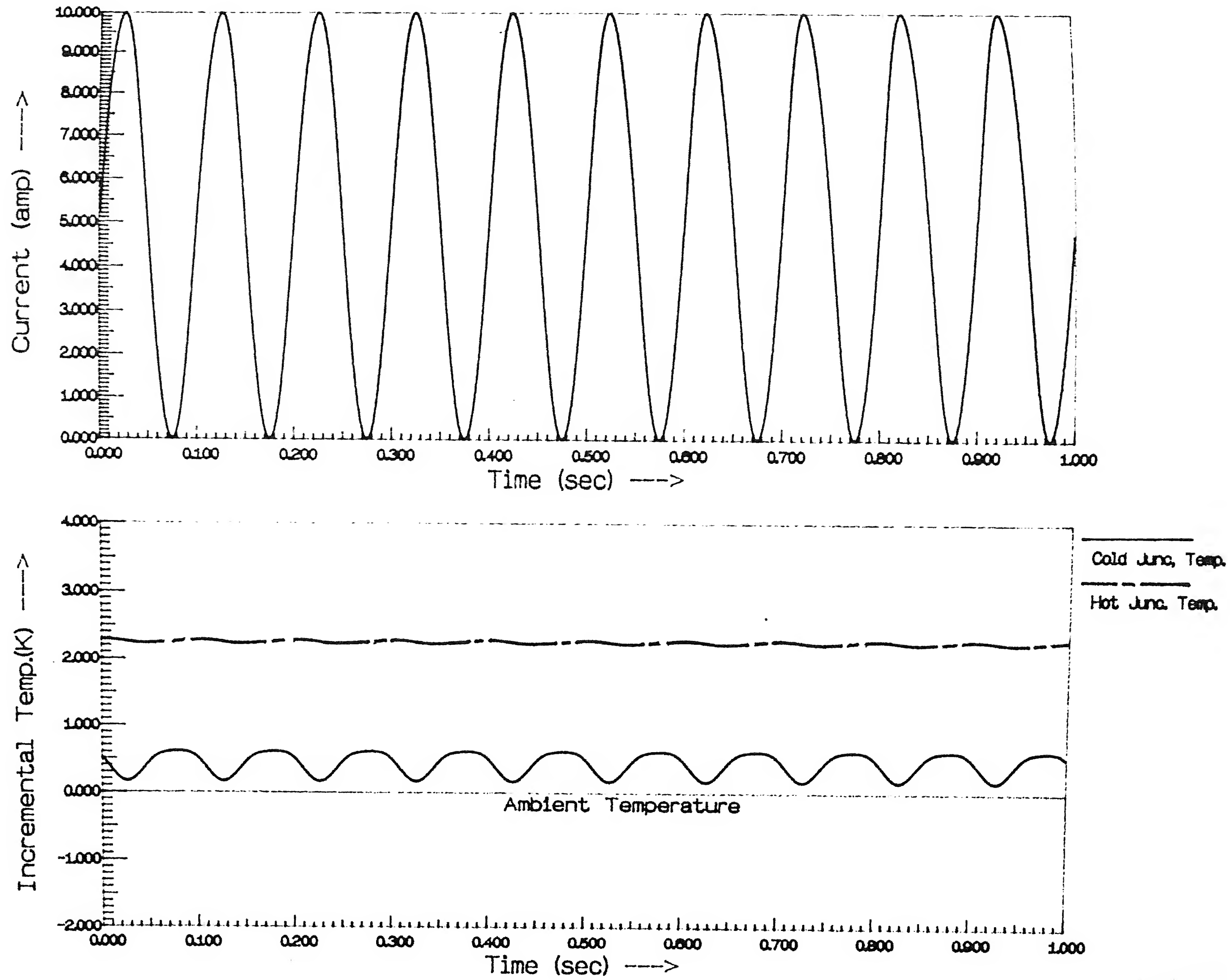


Fig. 5.23 Incremental Junction Temperatures for a Sinusoidal Current of Amplitude 5.0 A and Frequency 10 Hz. with a Constant Power Input of 0.5 watts.

TABLE - 5.2

PHASE DIFFERENCE BETWEEN CURRENT AND TEMPERATURE FOR VARIOUS FREQUENCIES

CURRENT (A)	FREQUENCY (Hz)	PHASE DIFFERENCE (DEG.)
$I_{01} = I_0 = 0.5$	0.1	162°
	1.0	173°
	10.0	180°
$I_{01} = I_0 = 5.0$	0.1	126°
	1.0	162°
	10.0	180°

CASE - ii(a) : Step change in the power input at the cold junction with the current kept constant at 1.0 A.

A step power input is given to the system operating initially under steady state conditions of current flow of 1.0 A and initial power input of 0.5 W. Five values of the power input are considered from 0.1 W to 0.5 W. The incremental cold junction and hot junction temperatures are shown plotted against time in Figs.5.24 through 5.28.

Both the cold and the hot junction temperatures increase as a result of the additional heat load. The cold junction temperature however increases more rapidly than the hot junction temperature. Both the junctions finally attain new steady state values.

The cold junction heats up almost immediately as a result of the additional power input. As the cold junction temperature increases the conduction of heat from the hot junction to the cold junction decreases and so the hot junction temperature also increases. Due to a finite heat capacitance (ie. ability to store heat) , of the thermocouple elements, the hot junction temperature increases at a slower rate as compared to the cold junction temperature.

As the step power input increases the final steady state temperatures of both the cold and hot junctions increase, but the *temperature difference* between the two ends decreases. Thus more heat is now being pumped across the thermoelectric couple but across a smaller temperature difference.

The incremental cold and hot junction temperatures for the various step power inputs have been plotted in Figs. 5.29 and 5.30

respectively. Fig. 5.29 shows how the incremental cold junction temperature increases as the step power input is increased. Similarly Fig.5.30 depicts the response of the hot junction to increasing values of the step power input. The plots clearly illustrate the various trends discussed above.

The transient response of the thermoelement for increasing power values, at various instants of time, has been depicted in Figs. 5.31 and 5.32. Figs. 5.31(a) to (l) show that the incremental cold junction temperature increases *linearly* with the thermal load at the cold junction. Figs. 5.32(a) to (l) show a similar trend for the hot junction temperature.

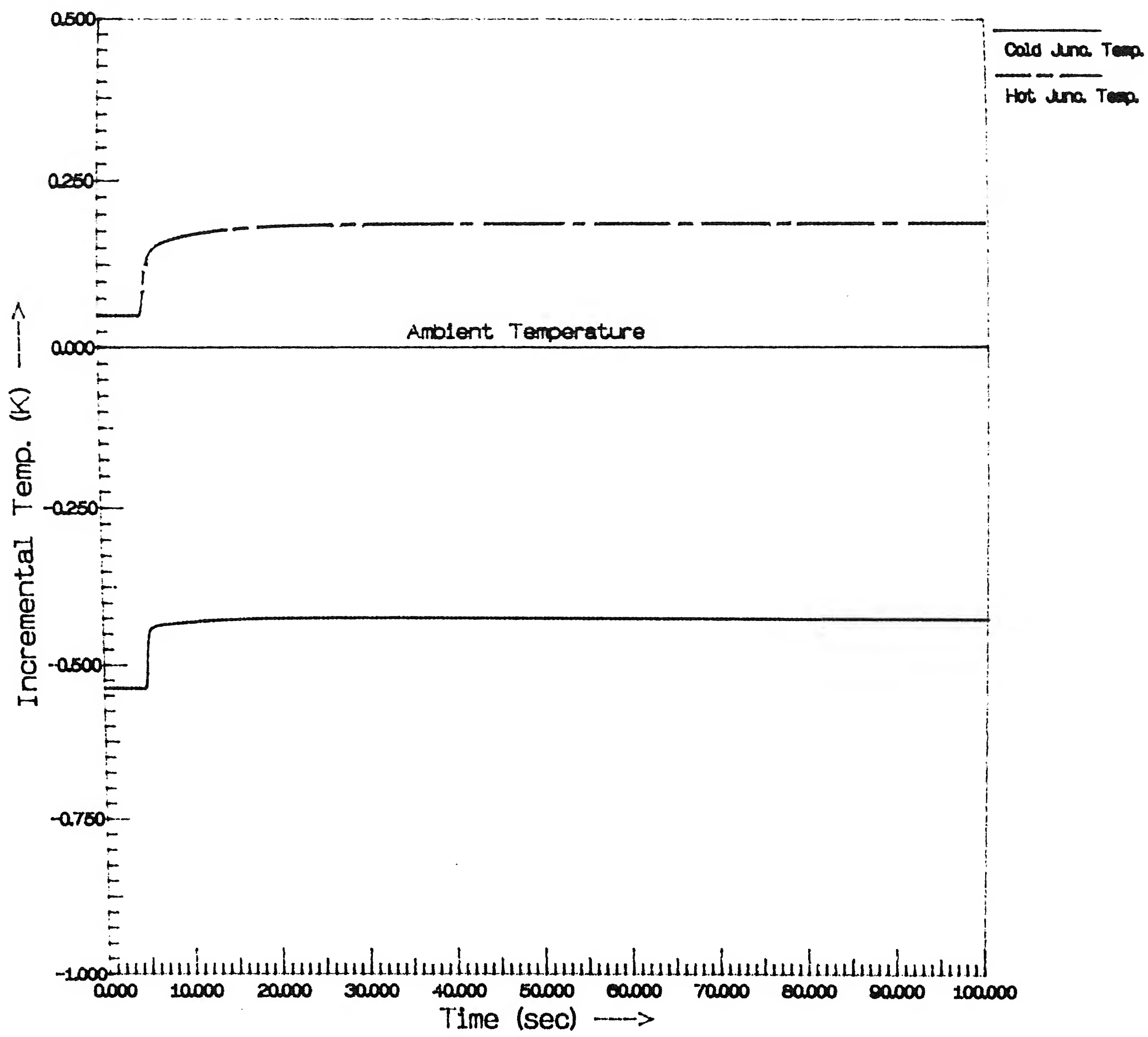


Fig. 5.24 Incremental Junction Temperatures for a Step Power Input of 0.1 W and Constant Current of 1.0 A.

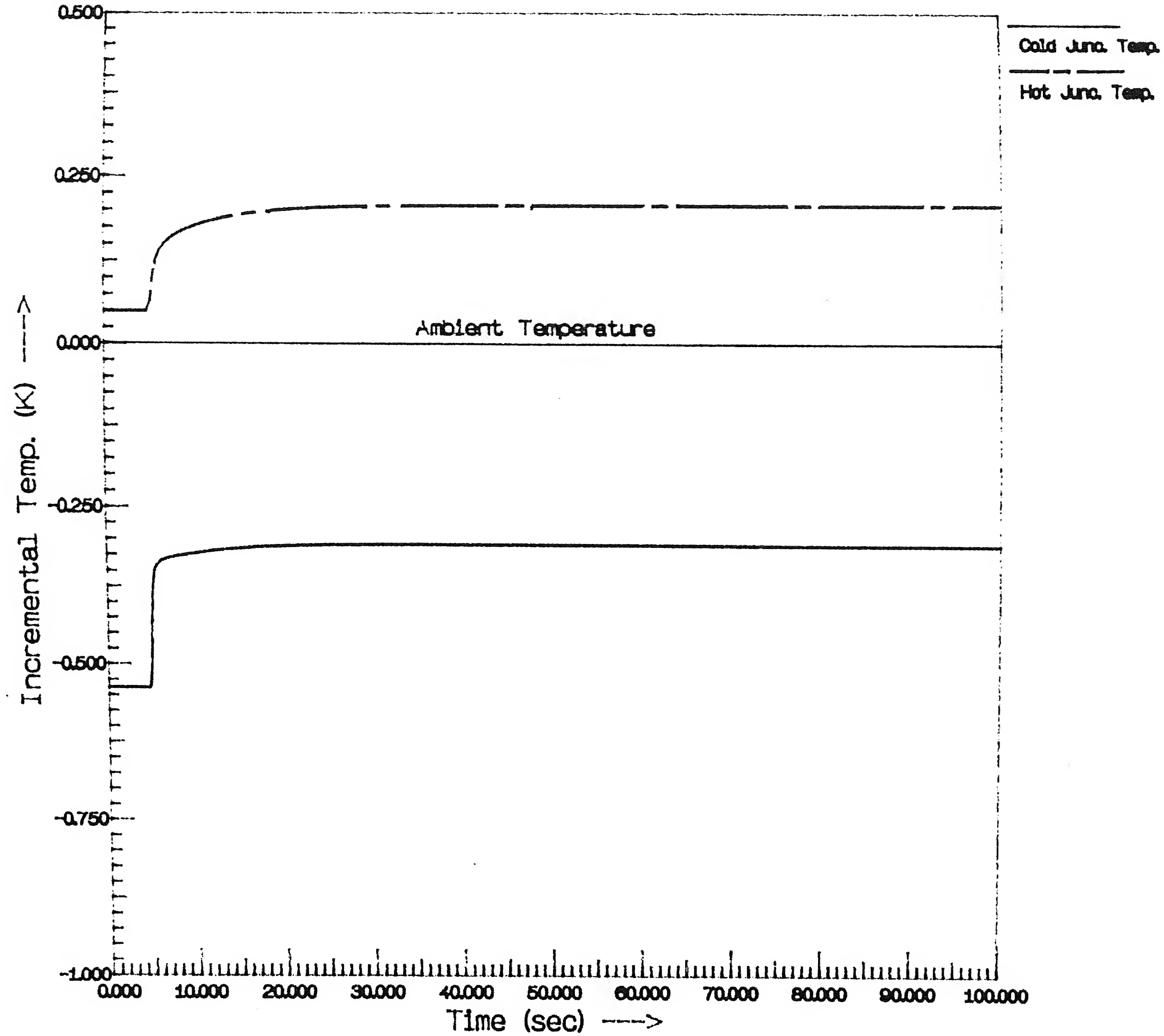


Fig. 5.25 Incremental Junction Temperatures for a Step Power Input of 0.2 W and Constant Current of 1.0 A.

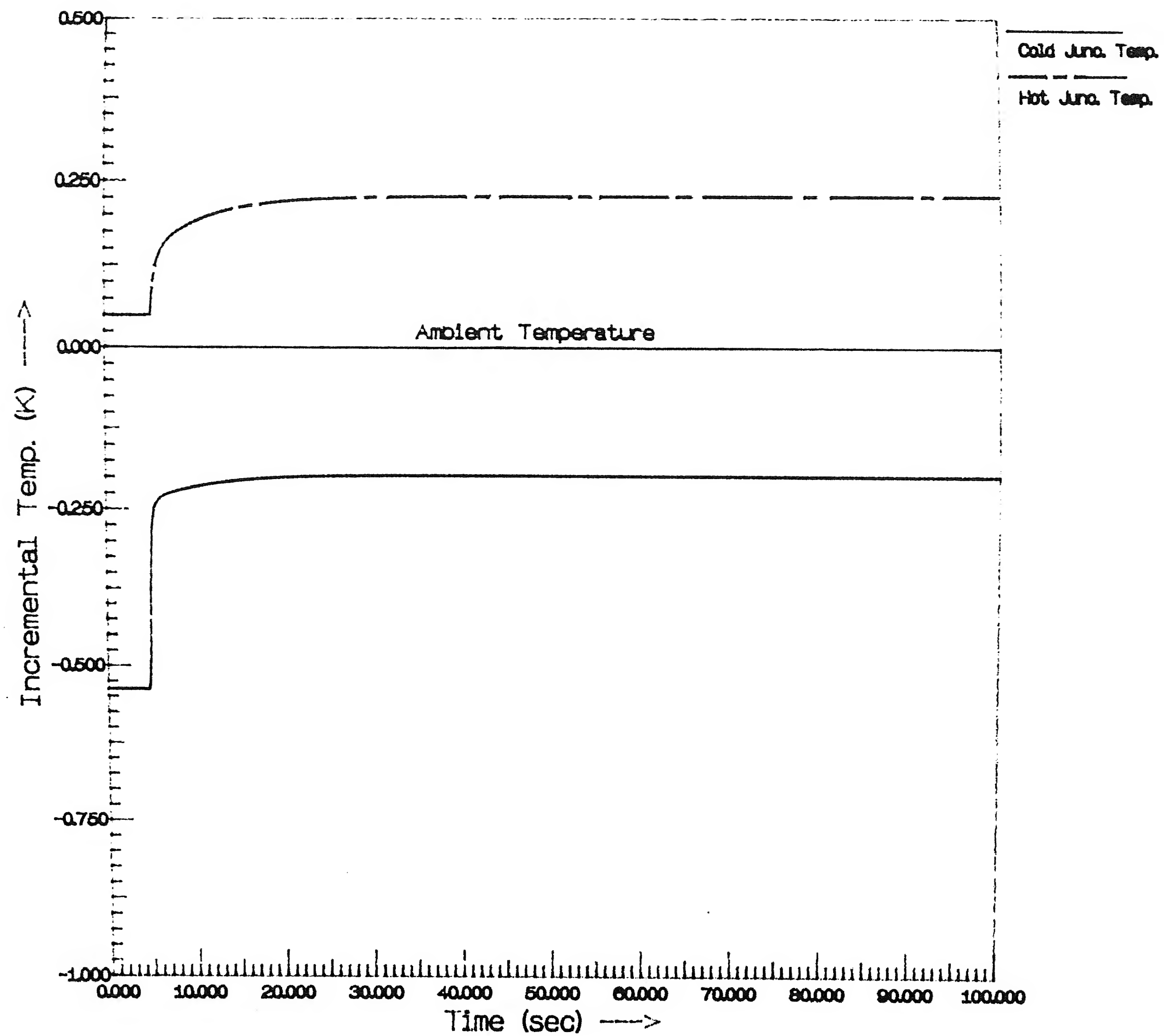


Fig. 5.26 Incremental Junction Temperatures for a Step Power Input of 0.3 W and Constant Current of 1.0 A.

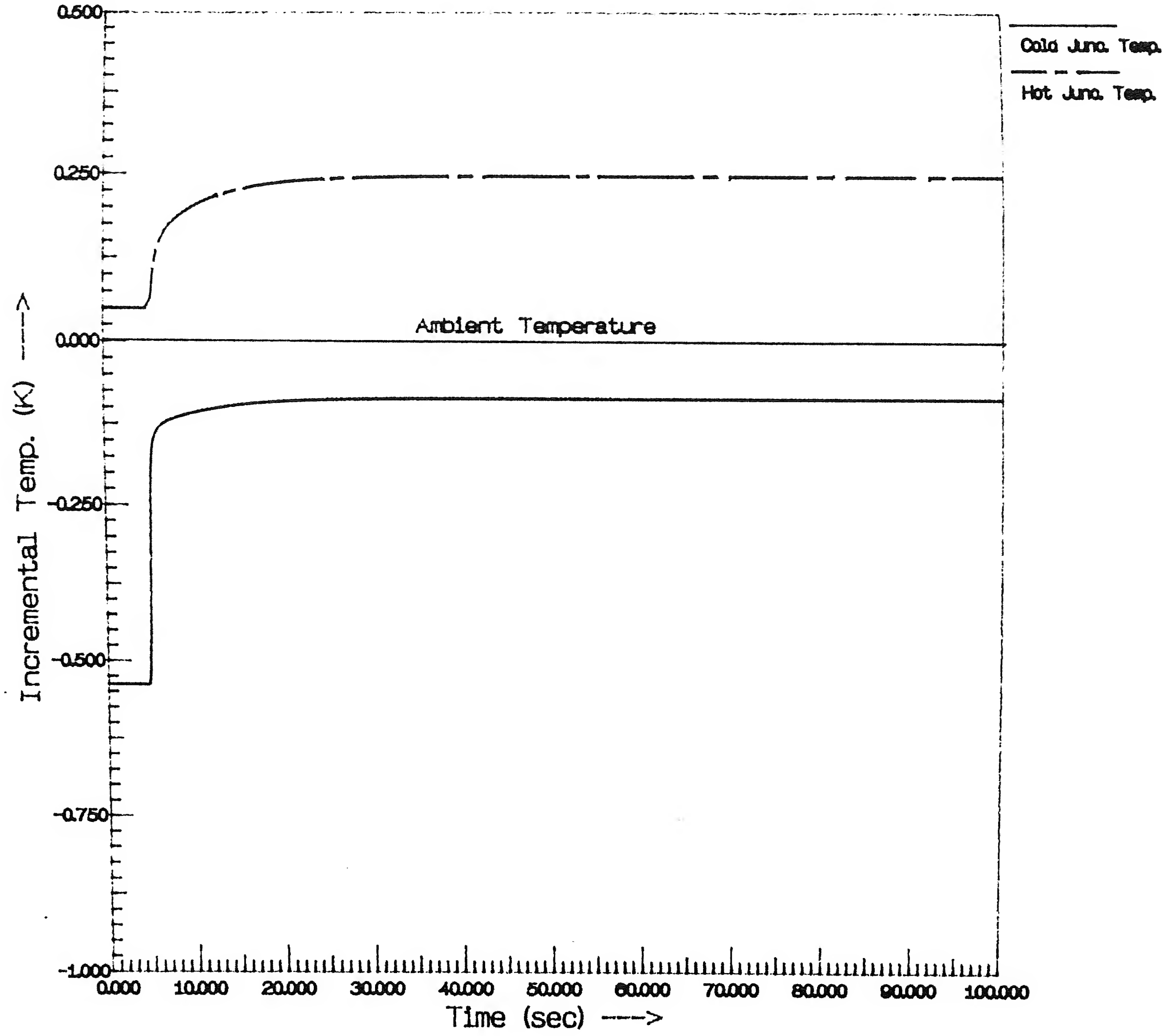


Fig. 5.27 Incremental Junction Temperatures for a Step Power Input of 0.4 W and Constant Current of 1.0 A.

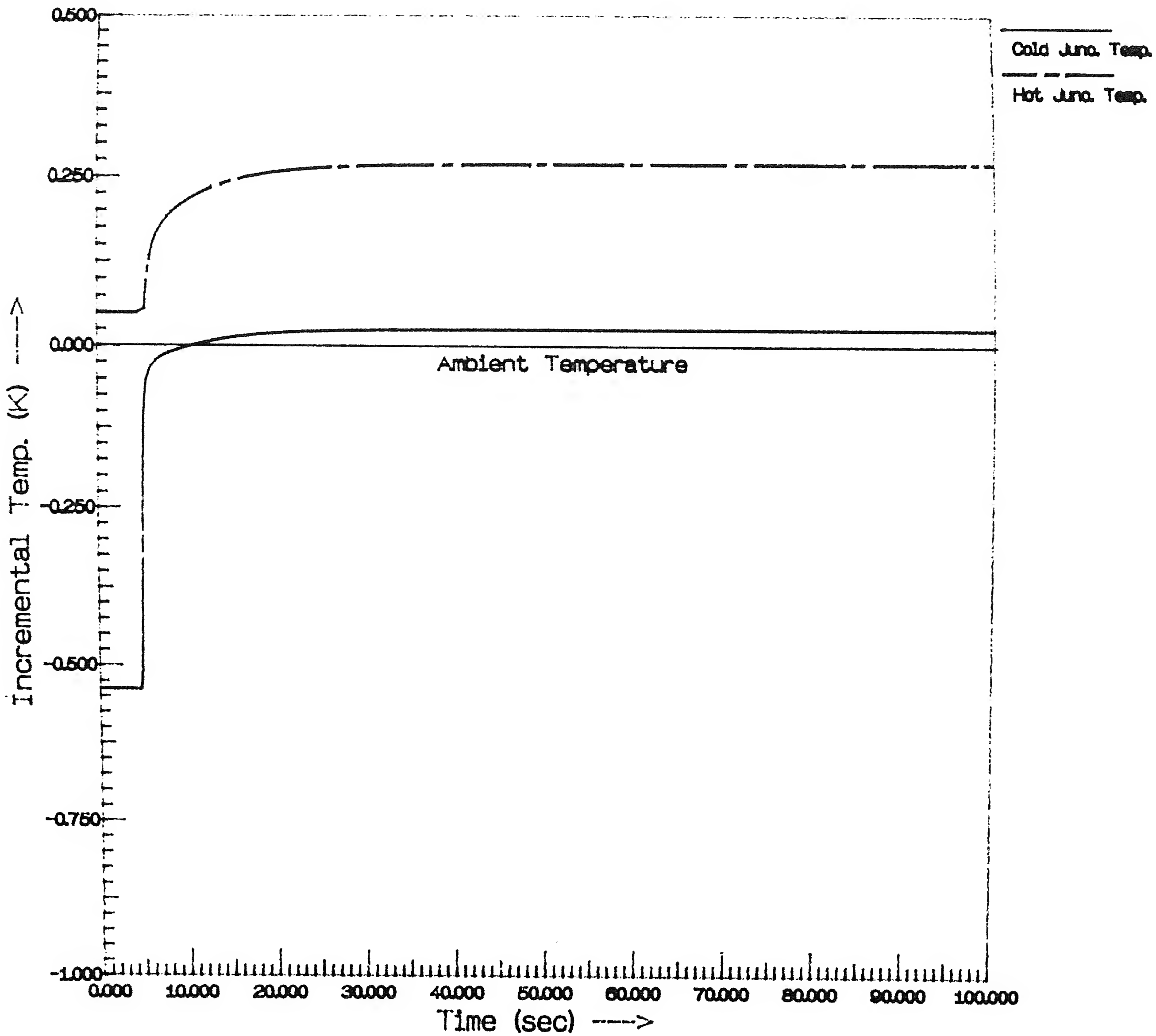


Fig. 5.28 Incremental Junction Temperatures for a Step Power Input of 0.5 W and Constant Current of 1.0 A.

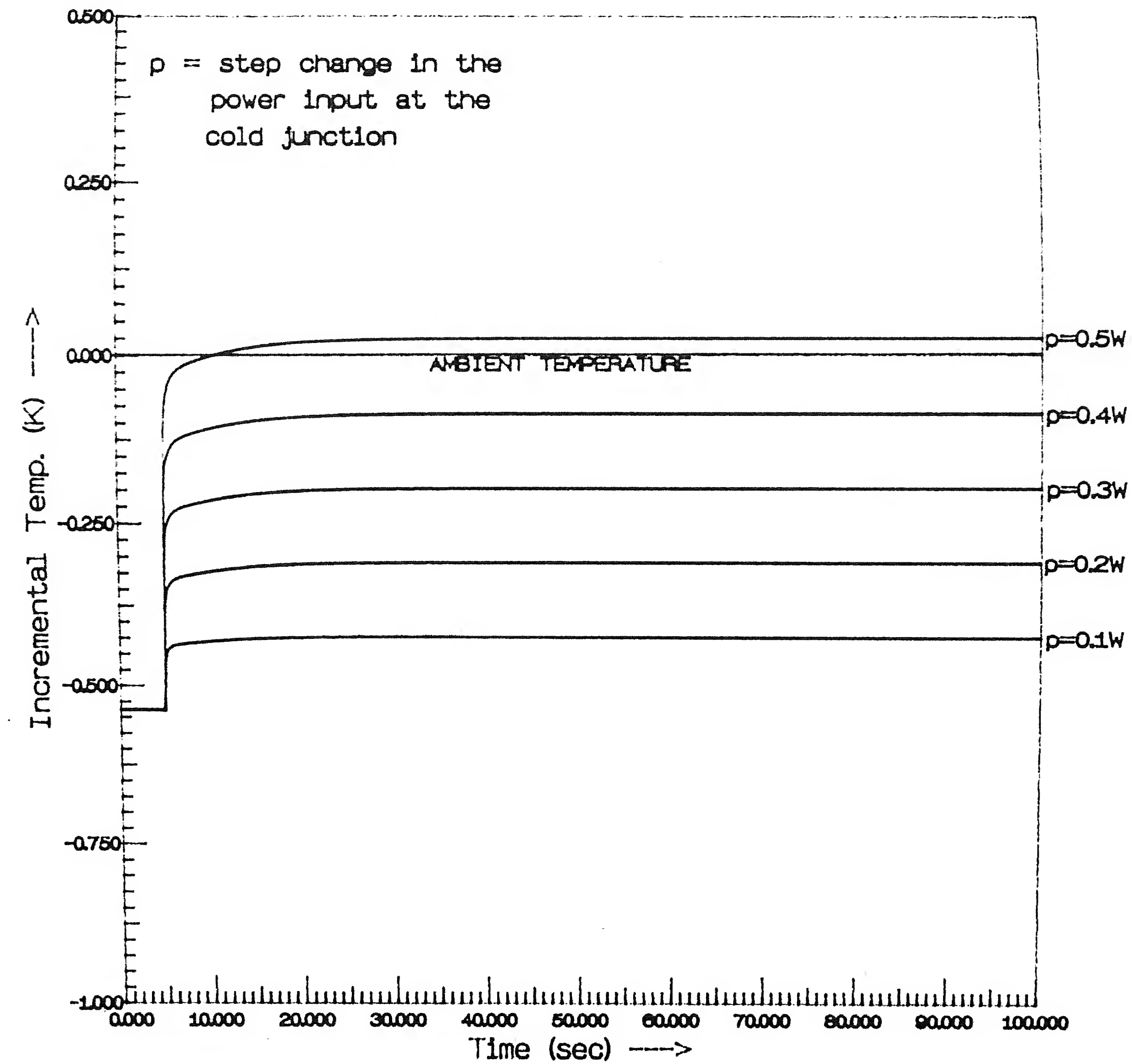


Fig. 5.29 Comparison of the Incremental Cold Junction Temperatures for Increasing Step Power Inputs.

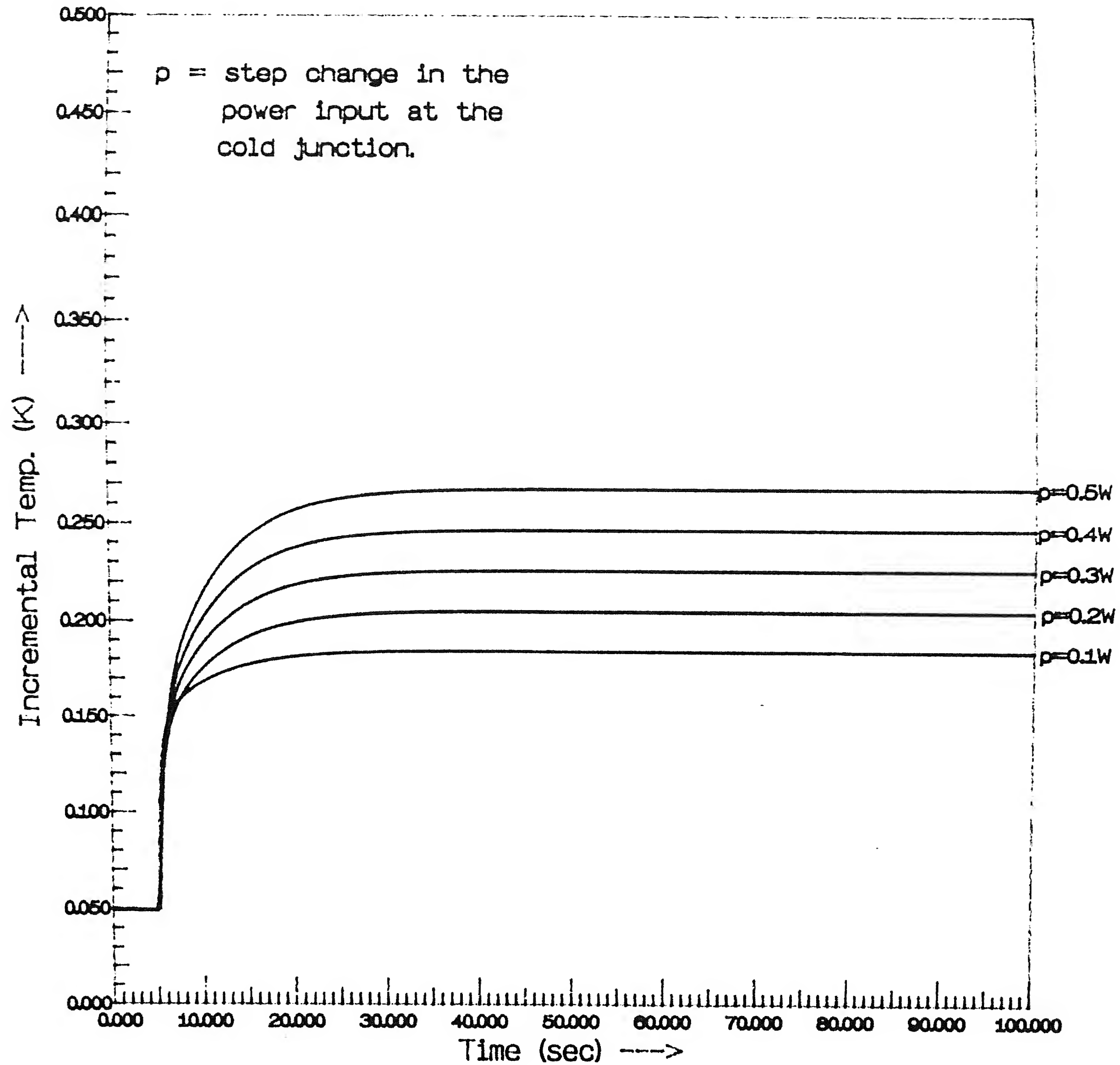


Fig. 5.30 Comparison of the Incremental Hot Junction Temperatures for Increasing Step Power Inputs.

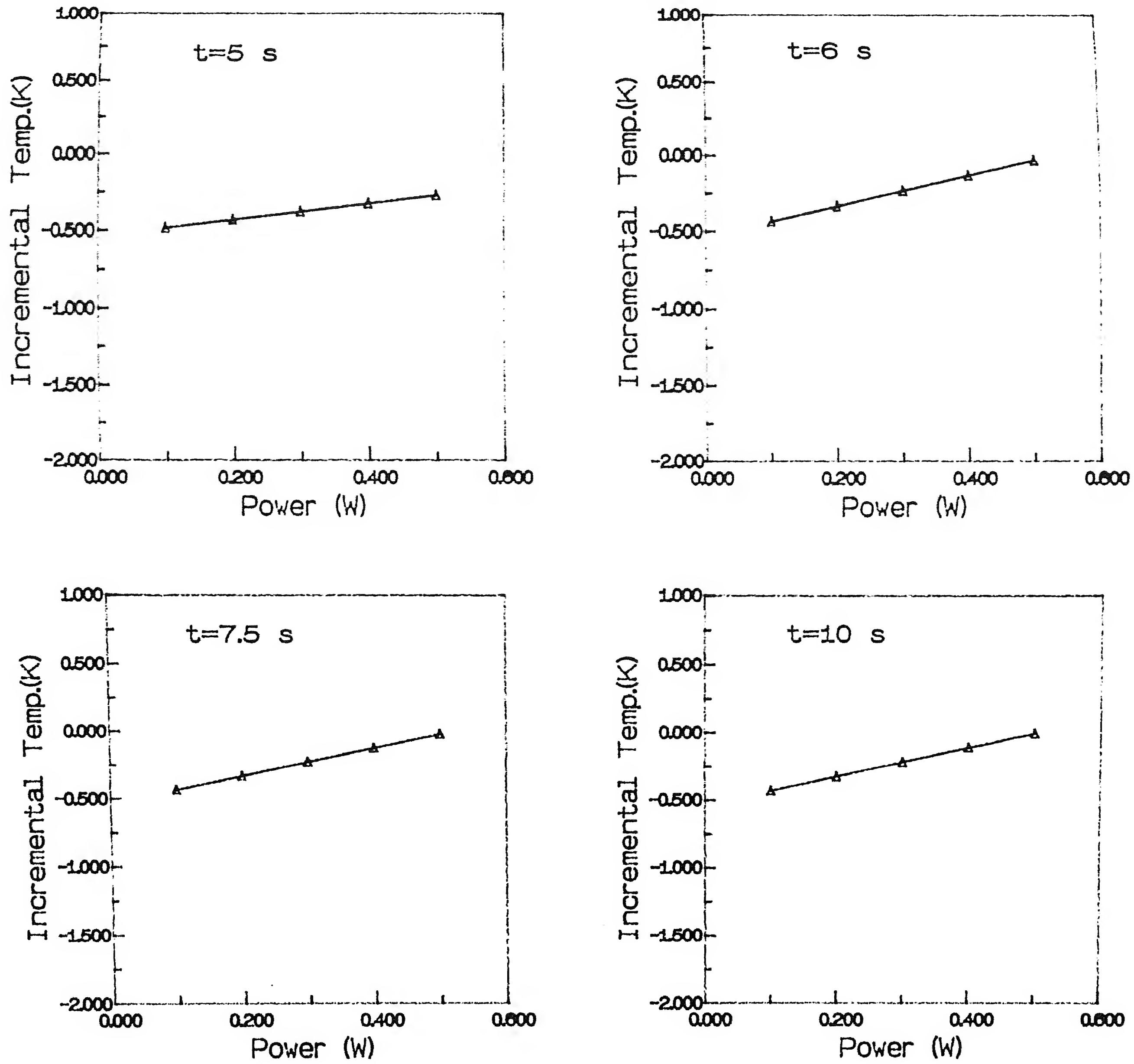


Fig. 5.31 (a) Incremental Cold Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

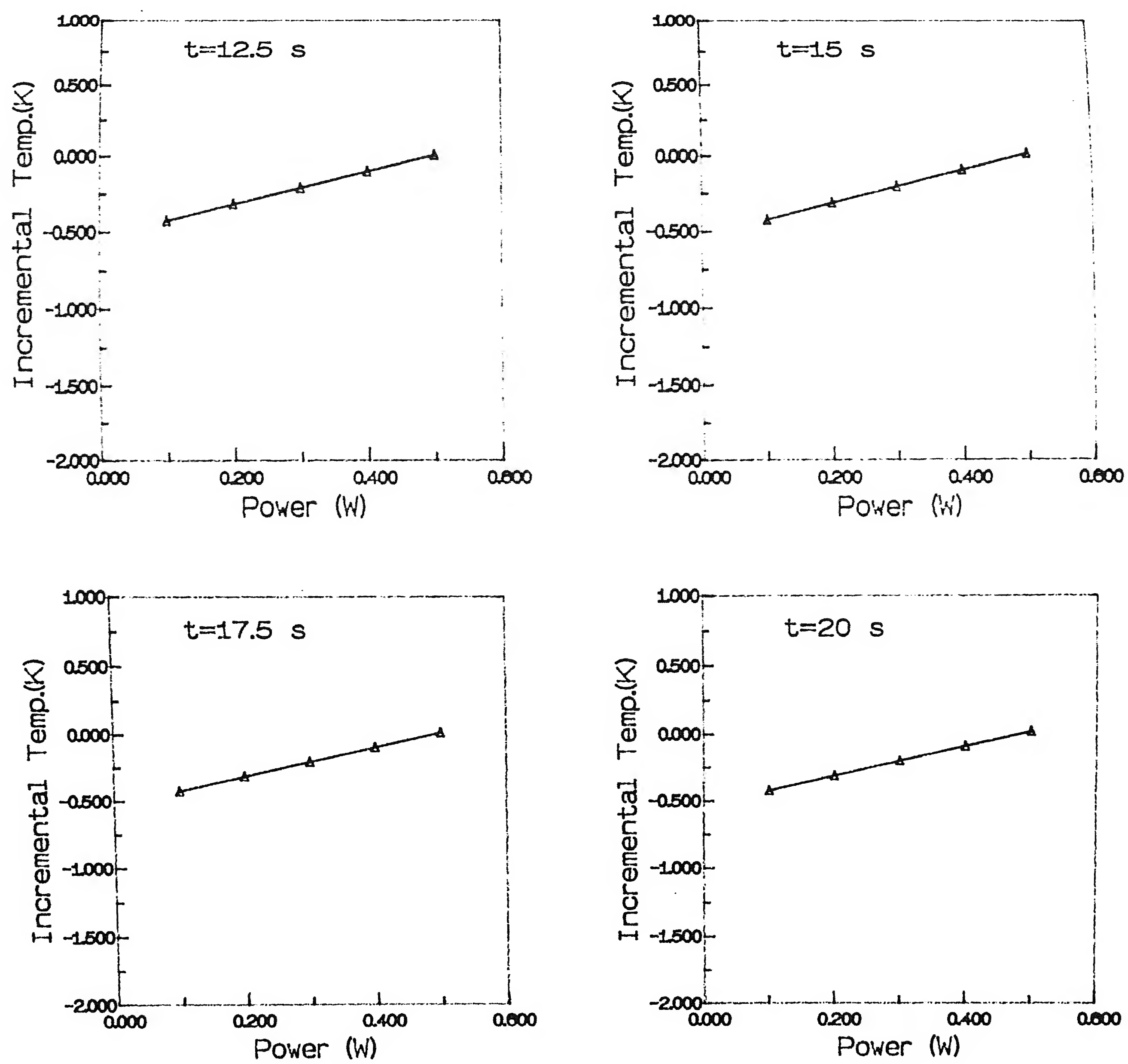


Fig. 5.31 (b) Incremental Cold Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

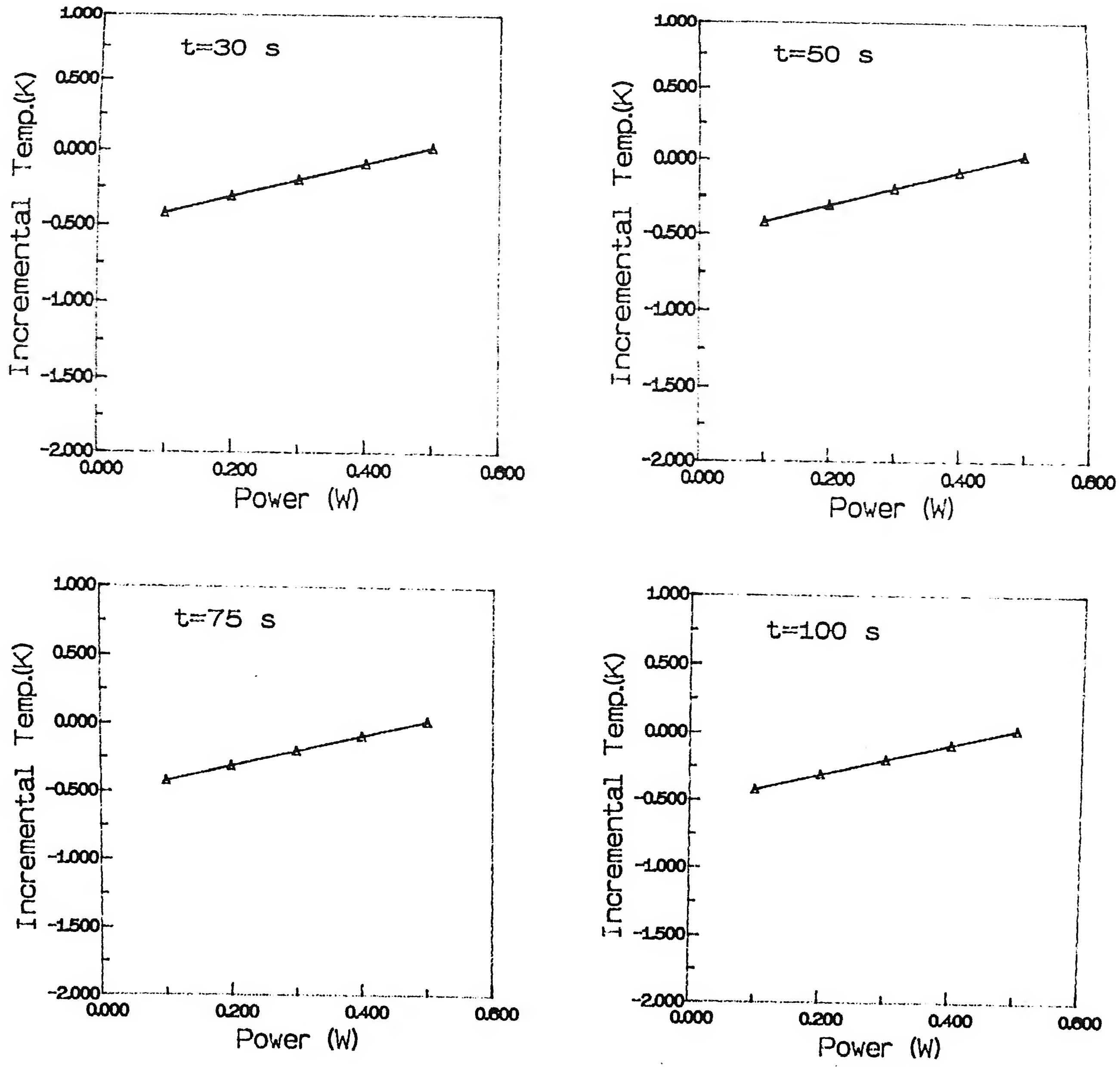


Fig. 5.31 (c) Incremental Cold Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

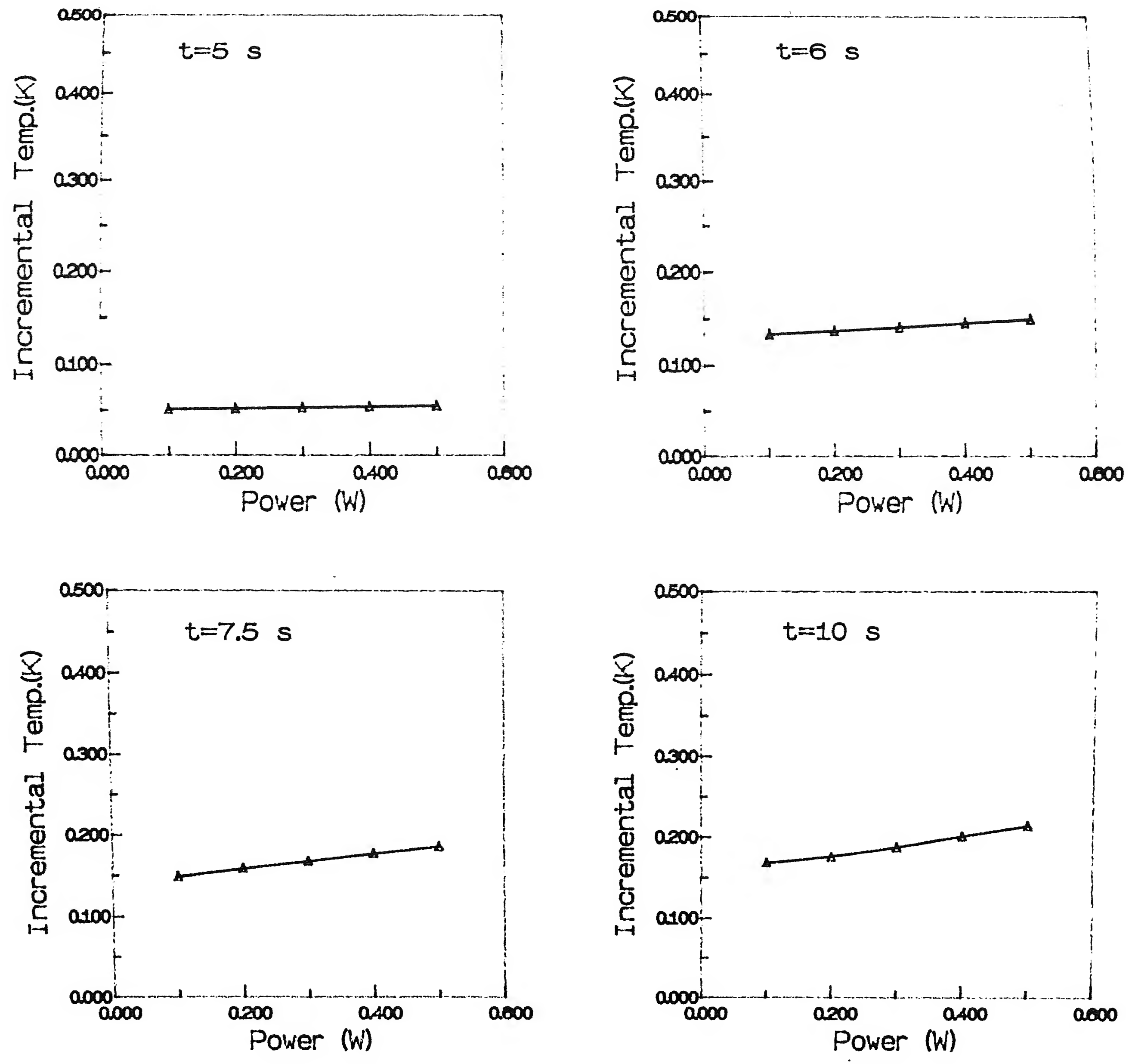


Fig. 5.32 (a) Incremental Hot Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

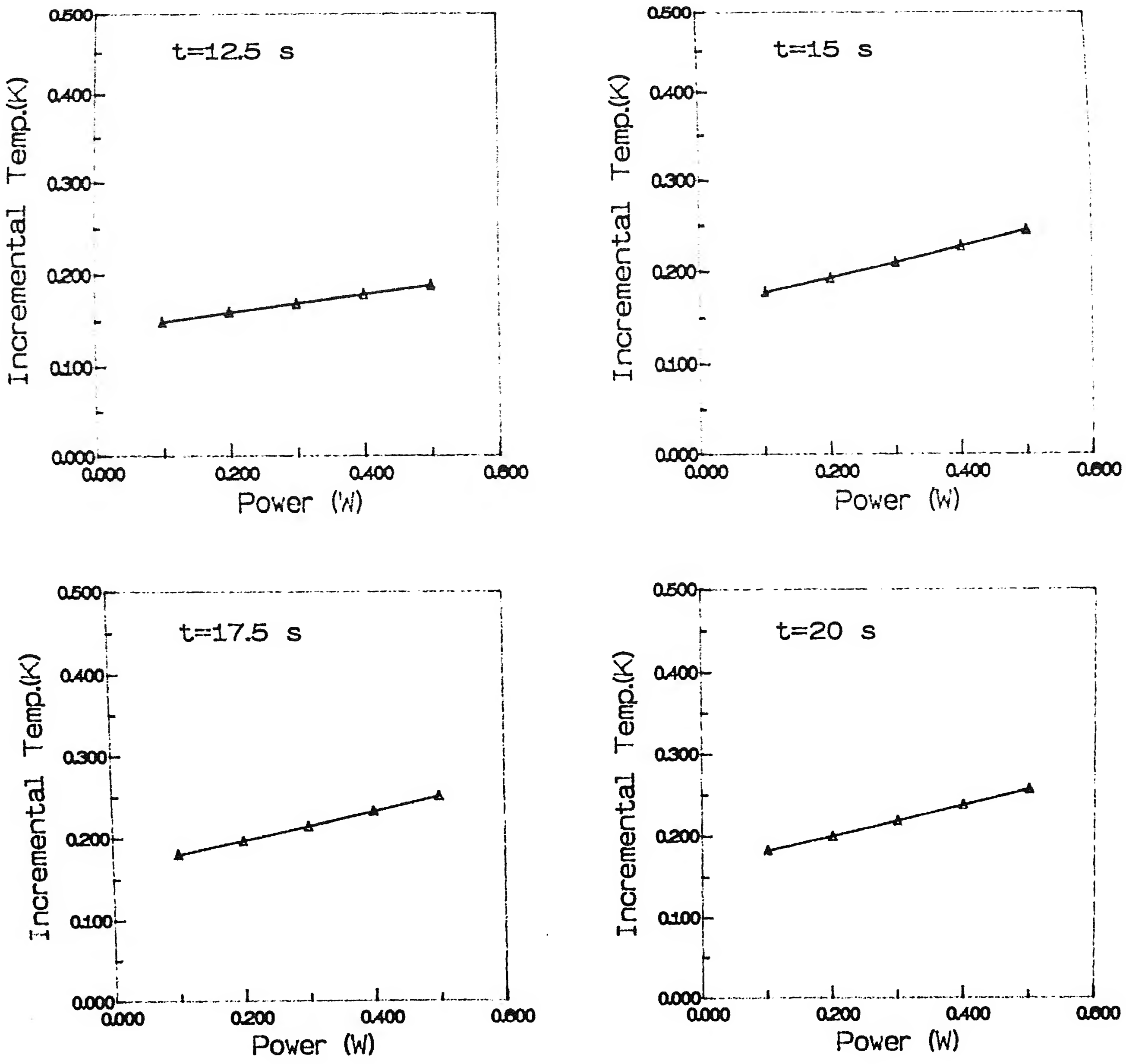


Fig. 5.32 (b) Incremental Hot Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

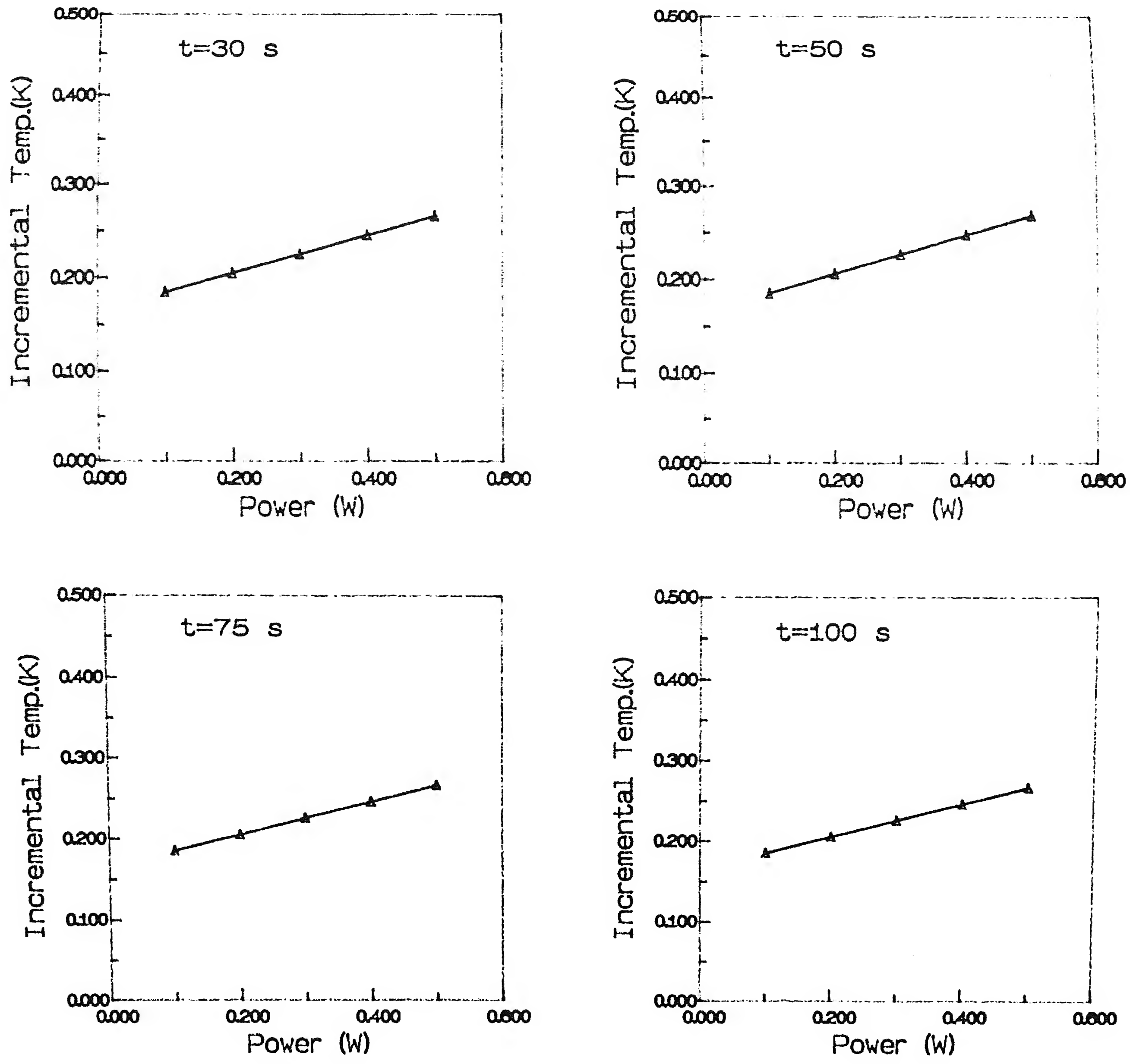


Fig. 5.32 (c) Incremental Hot Junction Temperatures for Increasing Step Power Inputs , at Various Instants of Time .

CASE - ii(b) : Sinusoidal change in the power input at the cold junction with the current kept constant at 1.0 A.

The system response is considered for a sinusoidal power input given to a thermocouple initially operating at a constant current of 1.0 A and a constant power input of 0.5 W. The sinusoidal power input is of the form :

$$p = p_0 . \sin(\omega . t)$$

where p_0 is the amplitude of the sinusoidal power input and is taken to be equal to 0.1 W. Three values of the frequency, ω, are considered - viz. 0.1 Hz., 1.0 Hz. and 10.0 Hz. The incremental cold and hot junction temperatures are shown plotted against time in Figs. 5.29 through 5.31. The power input is also plotted above the temperatures in each case.

The plots depict the phase behaviour of the device under a sinusoidal power input. The hot and cold junction temperatures are always in phase with one another and with the power input. The temperatures also have the same frequency which is equal to the frequency of the sinusoidal power input. The temperatures are now in phase with the sinusoidal power input because the temperature is a linear function of the power input.

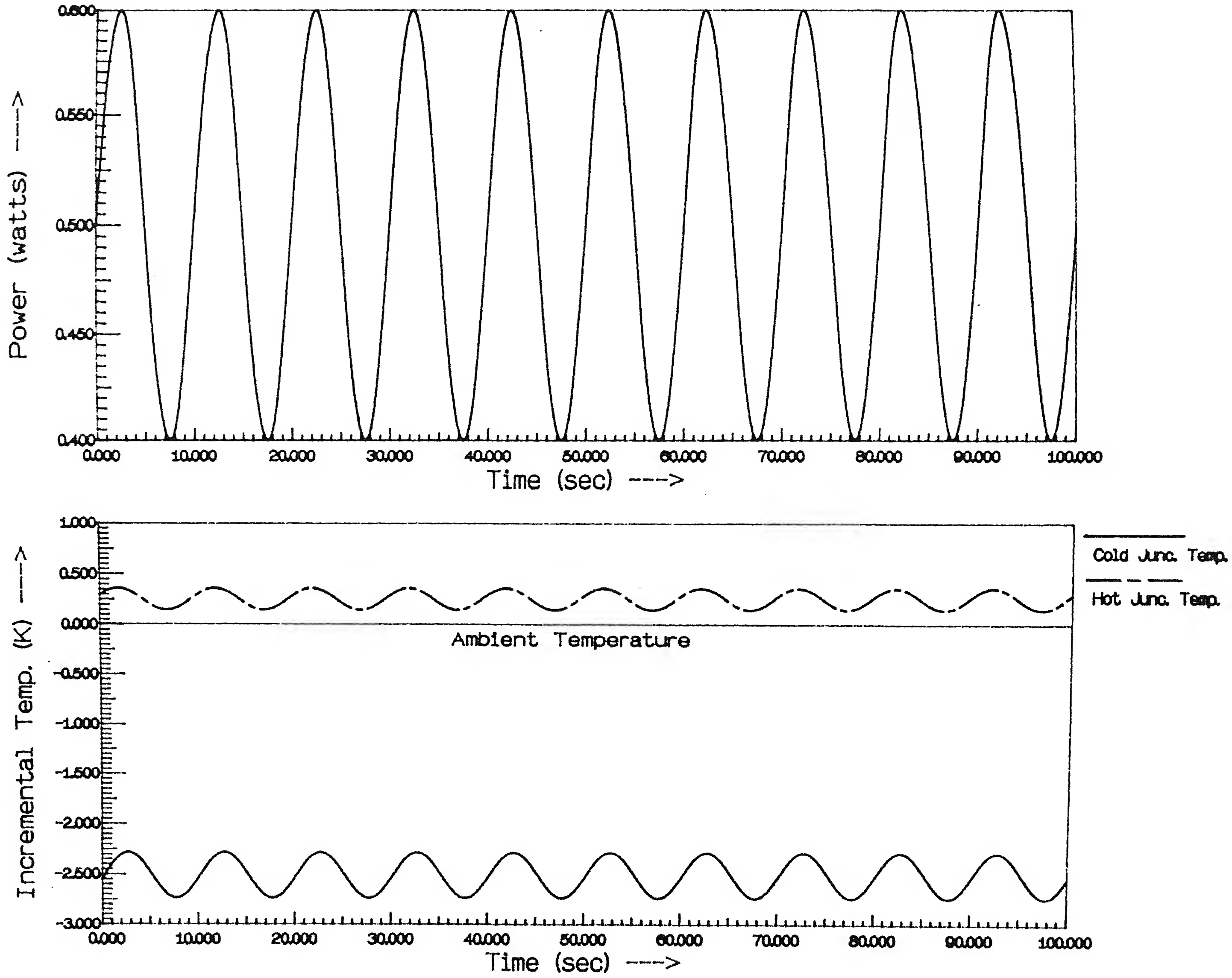


Fig. 5.33 Incremental Junction Temperatures for a Sinusoidal Power Input of Amplitude 0.1 W and Frequency 0.1 Hz. with a Constant Current of 1.0 A.

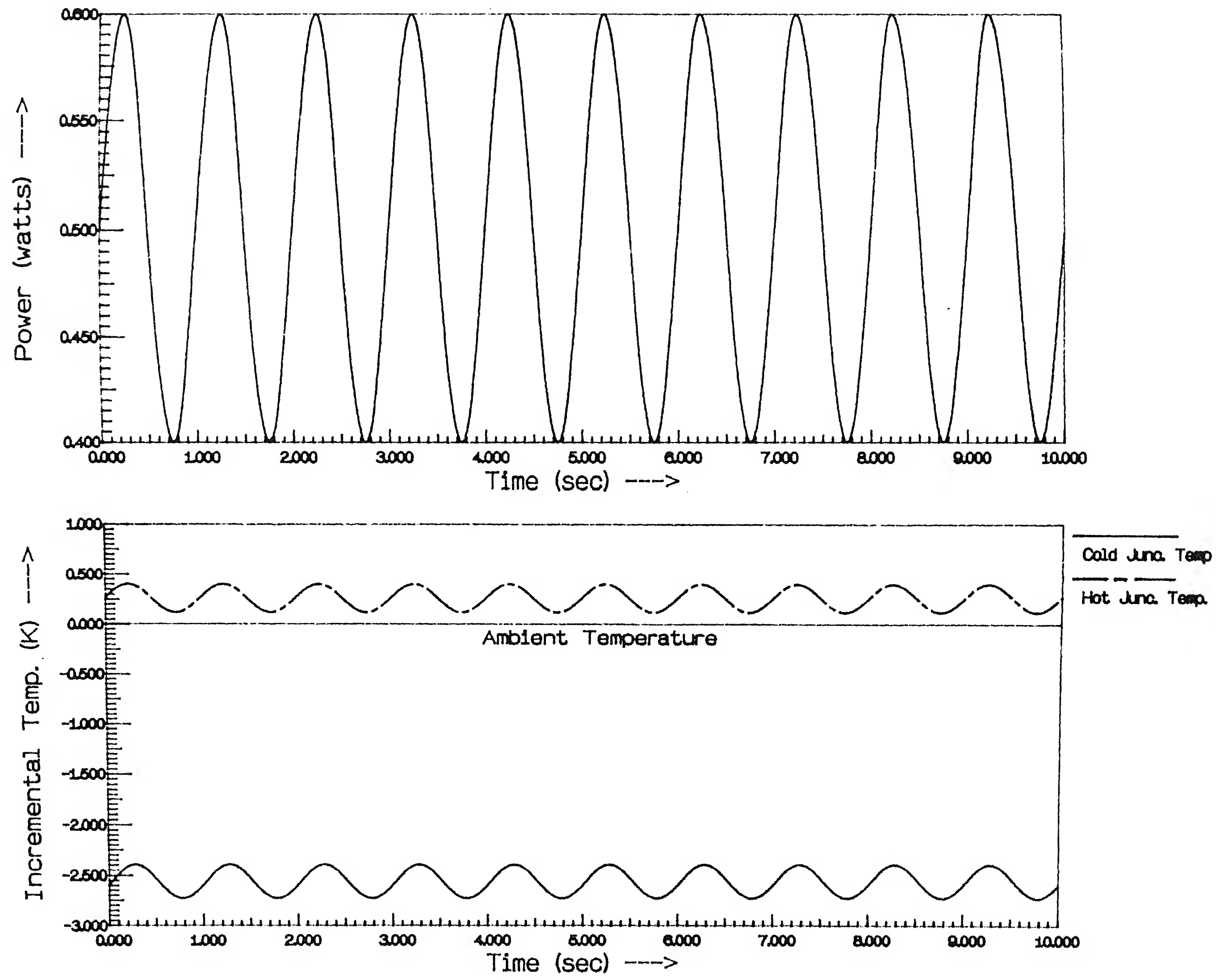


Fig. 5.34 Incremental Junction Temperatures for a Sinusoidal Power Input of Amplitude 0.1 W and Frequency 1.0 Hz. with a Constant Current of 1.0 A.

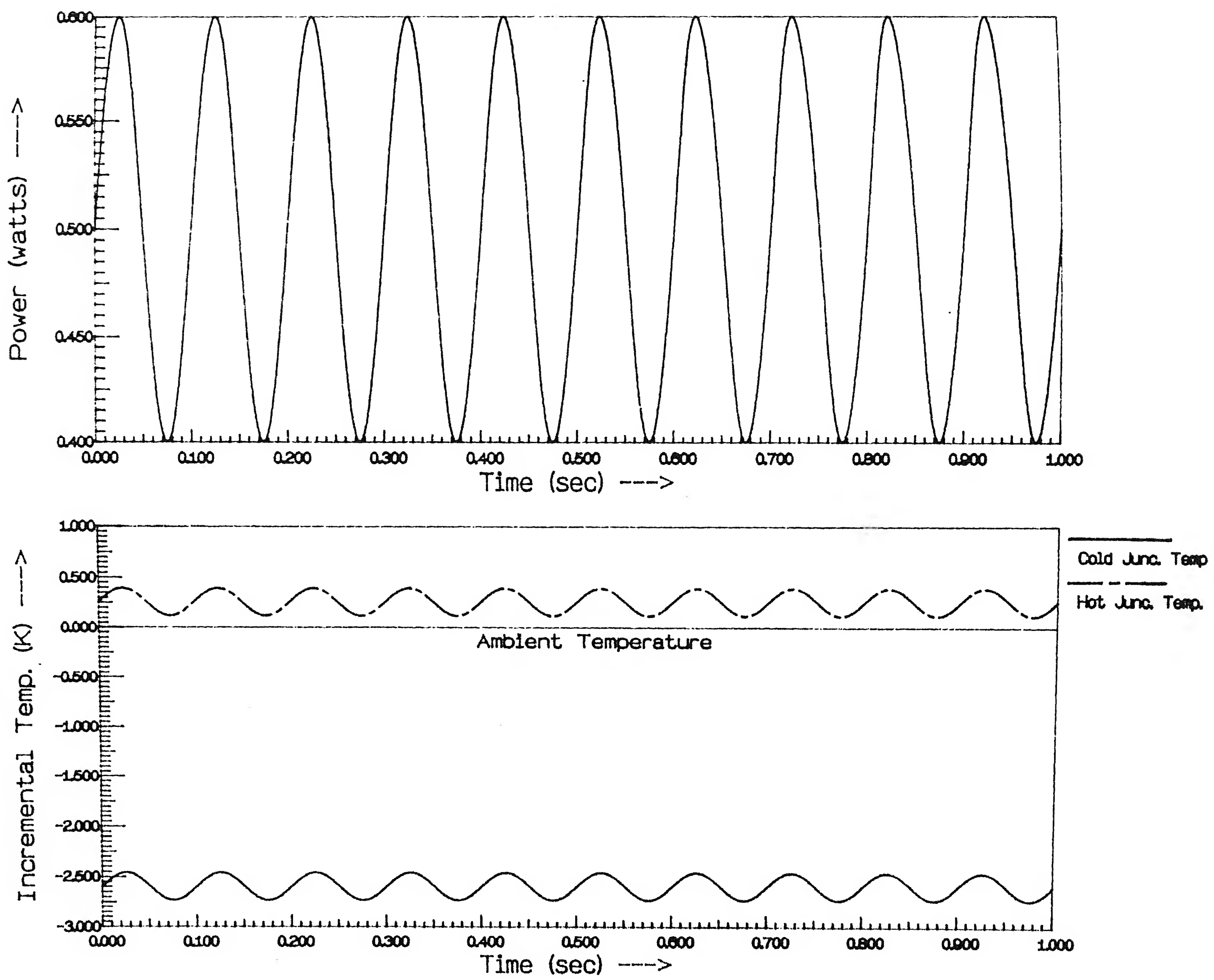


Fig. 5.35 Incremental Junction Temperatures for a Sinusoidal Power Input of Amplitude 0.1 W and Frequency 10 Hz. with a Constant Current of 1.0 A.

5.3 Conclusions :

The transient behaviour of a thermoelectric cooling device has been thoroughly investigated in the present work. The transient response of the device to a change in current is very important from the point of view of the controllability of the device, as current is the principal controlling parameter in the operation of the device.

The following conclusions can be drawn from the analysis :

(i) There is an optimum current at which the steady state temperature drop of the cold junction is maximum. For currents larger than the optimum value the steady state temperature of the cold junction increases, instead of decreasing, as the current is increased.

(ii) For currents larger than the optimum value there is an initial temperature drop at the cold junction which increases with current. This suggests that pulsed current inputs much larger than the optimum value could be used to produce temperature drops far greater than those achieved with steady currents. The current pulses must be timed so as to produce the additional temperature drop as soon as the minimum temperature due to the previous pulse is produced.

(iii) An increase in the thermal load causes the cold junction temperature to increase linearly. For currents less than the optimum value, an increase in the control current causes a drop in the cold junction temperature. But if the current is greater than the optimum value then as steady state is reached the cold junction temperature increases due to Joulean heat generation. Thus the only solution in this case would be to have additional

cooling elements in parallel, to take up the increased thermal load.

(iv) The cold junction temperature shows a sinusoidal response when the thermal load at the cold junction varies sinusoidally. Again for a sinusoidal current the cold junction temperature also varies sinusoidally, but there is a phase shift between the current and the temperature in this case. Therefore for a sinusoidal thermal load the cold junction temperature can be controlled by a sinusoidal current if the phase difference is taken into account. It must be noted here that in practical situations the thermal load will rarely be perfectly sinusoidal. However any periodic load can always be written as a Fourier series which would amount to the superposition of a number of sinusoidal loads. The control current would accordingly have to be the sum of a similar Fourier series with the phase difference added to each of the terms of the series.

5.4 Suggestions for Future Work :

The present study is a step towards the design and development of practical thermoelectric cooling devices in which temperature controllability is an important factor. In this work the temperature response characteristics of only a single element have been considered. However thermoelectric devices rarely consist of a single stage. Usually two or three stages are cascaded to achieve the desired temperature drop. The dynamic behaviour of such a system needs to be studied along similar lines, as the control aspects of this type of a device are of interest in the design of practical thermoelectric cooling

devices.

Today a large number of semiconductor materials are available. Work needs to be done to investigate the effects of their property values on the behaviour of thermoelectric devices and to find the optimal operating conditions for each material.

Work is also required to investigate as to how a semiconductor element responds to a pulsed current and to find the time period of the pulses that would give the largest drop in temperature at the cold junction. The control aspects of the element under a sinusoidal current also need to be studied further, to determine a relationship for the phase difference between the cold junction temperature and the current. These results would be extremely important for the design of thermoelectric cooling devices that have to operate under a periodic thermal load.

Lastly the results established here and in any future work need to be verified through extensive experimentation.

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APPENDIX - A

DERIVATION OF THE KELVIN RELATIONS

The Kelvin relations may be derived by applying the laws of thermodynamics to the simple circuit shown in the figure below:

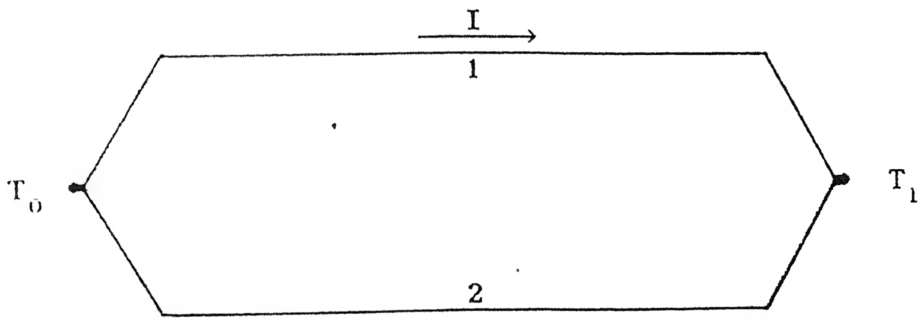

Fig. A.1. Simple Thermoelectric Circuit

A current I passes round the circuit which consists of conductors 1 and 2 with junctions at temperatures T_0 and T_1. From the principle of conservation of energy the heat generated must be equal to the consumption of electrical energy. If the current is small enough, Joule heating may be neglected. Thus,

$$\left\{\left(\Pi_{12}\right)_1 - \left(\Pi_{12}\right)_0\right\} + \int_{T_0}^{T_1}\left(\tau_1 - \tau_2\right).I.dT = \int_{T_0}^{T_1}\alpha_{12}.I.dT \qquad \ldots(A.1)$$

Differentiating, we get :

$$\frac{d\Pi_{12}}{dT} + \tau_1 - \tau_2 = \alpha_{12} \qquad \ldots(A.2)$$

In order to obtain a second relation between the coefficients it is necessary to apply the second law of thermodynamics to the above thermoelectric circuit. In order that this step should be valid it is essential that the process should be reversible. Though the thermoelectric effects are reversible, but they are accompanied by the irreversible processes of Joule heating and heat conduction. Thus, the application of reversible thermodynamics is not strictly justified. However the use of Onsager's reciprocal relations, which are based on irreversible thermodynamics, leads to the same conclusions[18], so we shall assume that there is no change of entropy round the circuit of Fig. A.1. Then,

$$\int_0^1 I.d\left(\frac{\Pi_{12}}{T}\right) + \int_{T_0}^{T_1}\left(\frac{\tau_1 - \tau_2}{T}\right).I.dT = 0 \qquad \ldots(A.3)$$

On differentiating it we get :

$$\frac{d\Pi_{12}}{dT} - \frac{\Pi_{12}}{T} + \tau_1 - \tau_2 = 0 \qquad \ldots(A.4)$$

Combining equations A.2 and A.4 we have :

$$\alpha_{12} = \frac{\Pi_{12}}{T}$$

or

$$\alpha_{12}.T = \Pi_{12} \qquad \ldots(A.5)$$

which is the first of the Kelvin relations. It relates the Seebeck and the Peltier coefficients. When applied to the two junctions of the generic thermoelectric device depicted in Fig. 4.1, it yields the following equations :

$$\alpha_1 . T_1 = \Pi_1 \qquad \ldots(4.12)$$

and

$$\alpha_0 . T_0 = \Pi_0 \qquad \ldots(4.13)$$

The second Kelvin relation connects the Seebeck and Thomson coefficients. Its derivation is based on the principal of conservation of energy. In section 4.3.3 it was shown that the overall energy balance for the generic thermoelectric device of Fig.4.1 results in the following equation :

$$V = \left[\left(\alpha_1 . T_1 - \alpha_0 . T_0 \right) - \int_{T_0}^{T_1} (\tau_1 - \tau_2) . dT \right]$$

$$- I . \left[\frac{1}{a_1} . \int_0^l \rho_1 . dx + \frac{1}{a_2} . \int_0^l \rho_2 . dx \right] \qquad \ldots(4.20)$$

The first Kelvin relation, equation A.5 has been used in the derivation of equation 4.20. If the electric current is zero we have :

$$V_0 = \alpha_1 . T_1 - \alpha_0 . T_0 - \int_{T_0}^{T_1} (\tau_1 - \tau_2) . dT \qquad \ldots(A.6)$$

where V_0 is the open-circuit terminal voltage. If equation A.6 is differentiated with respect to T_1 the result is :

$$\frac{dV_0}{dT_1} = T_1 . \frac{d\alpha_1}{dT_1} + \alpha_1 - (\tau_1 - \tau_2) \qquad \ldots(A.7)$$

since $\dfrac{dV_0}{dT_1} = \alpha_1$ by definition of the Seebeck coefficient, equation A.7 becomes :

$$\frac{d\alpha_1}{dT_1} = \frac{\tau_1 - \tau_2}{T_1} \qquad \text{...(A.8)}$$

and putting $\tau_1 - \tau_2 = \tau$, the net Thomson coefficient for the couple we get :

$$\frac{d\alpha_1}{dT_1} = \frac{\tau}{T_1} \qquad \text{...(A.9)}$$

which is the Second Kelvin relation for the junction at $x = 1$. If equation A.6 is differentiated with respect to T_0 and if the definition of α_0 is used, namely $\alpha_0 = -(dV_0/dT_0)$, the result is :

$$\frac{d\alpha_1}{dT_1} = \frac{\tau}{T_1} \qquad \text{...(A.10)}$$

which is the second Kelvin relation for the junction at $x = 0$.

Although equations A.9 and A.10 have been obtained by setting $I = 0$, they are valid when $I \neq 0$, since they are relations among the parameters of the thermoelectric device, and those parameters do not depend on the current.

from the above we have :

$$\frac{T^{n+1}_{1,i} - T^{n}_{1,i}}{\Delta t} = \left[\frac{k_1}{c_1 . \Delta x} + \frac{\tau_1 . I}{2 . c_1 . a_1} \right] . \frac{1}{\Delta x} . T^{n}_{1,i+1} + \left[- \frac{2 . k_1}{c_1 . \Delta x^2} + \frac{\eta_1 . I^2}{c_1 . a_1^2} \right] . T^{n}_{1,i}$$
$$+ \left[\frac{k_1}{c_1 . \Delta x^2} - \frac{\tau_1 . I}{2 . c_1 . a_1 . \Delta x} \right] . T^{n}_{1,i-1} + \frac{\rho_{10} . I^2}{c_1 . a_1^2}$$

ie.
$$T^{n+1}_{1,i} = \left[\frac{k_1}{\Delta x} + \frac{\tau_1 . I}{2 . a_1} \right] . \frac{\Delta t}{c_1 \Delta x} . T^{n}_{1,i+1}$$
$$+ \left[\left\{ - \frac{2 . k_1}{c_1 . \Delta x^2} + \frac{\eta_1 . I^2}{c_1 . a_1^2} \right\} . \Delta t + 1 \right] . T^{n}_{1,i}$$
$$+ \left[\frac{k_1}{\Delta x} - \frac{\tau_1 . I}{2 . a_1} \right] . \frac{\Delta t}{c_1 \Delta x} . T^{n}_{1,i-1} + \frac{\rho_{10} . I^2}{c_1 . a_1^2} . \Delta t \qquad \text{...(B.1)}$$

Again the equation for the temperature distribution in element 2, as derived in section 4.3.1 is :

$$k_2 . a_2 . \frac{\partial^2 T_2}{\partial x^2} - \tau_2 . I . \frac{\partial T_2}{\partial x} - c_2 . a_2 . \frac{\partial T_2}{\partial t} + \frac{\eta_2}{a_2} . I^2 . T_2$$
$$+ \frac{\rho_{20}}{a_2} . I^2 = 0 \qquad \text{...(4.24)}$$

rearranging it we have :

$$\frac{\partial T_2}{\partial t} - \frac{k_2}{c_2} . \frac{\partial^2 T_2}{\partial x^2} + \frac{\tau_2 . I}{c_2 . a_2} . \frac{\partial T_2}{\partial x} - \frac{\eta_2 . I^2}{c_2 . a_2^2} . T_2 - \frac{\rho_{20} . I^2}{c_2 . a_2^2} = 0$$

and on discretising it we get :

$$\frac{T^{n+1}_{2,i} - T^{n}_{2,i}}{\Delta t} - \frac{k_2}{c_2} . \frac{T^{n}_{2,i+1} - 2.T^{n}_{2,i} + T^{n}_{2,i-1}}{\Delta x^2} + \frac{\tau_2 . I}{c_2 . a_2} . \frac{T^{n}_{2,i+1} - T^{n}_{2,i-1}}{2.\Delta x} - \frac{\eta_2 . I^2}{c_2 . a_2^2} . T^{n}_{2,i} - \frac{\rho_{20} . I^2}{c_2 . a_2^2} = 0$$

where ,

$T^{n}_{2,i}$ denotes the temperature at the i^{th} point and the n^{th} time step in element 2, and so on,

the other symbols have their usual meanings.

Rearranging the above we get :

$$\text{ie. } T^{n+1}_{2,i} = \left[\frac{k_2}{\Delta x} - \frac{\tau_2 . I}{2.a_2} \right] \frac{\Delta t}{c_2 \Delta x} . T^{n}_{2,i+1} + \left[\left\{ - \frac{2.k_2}{c_2 . \Delta x^2} + \frac{\eta_2 . I^2}{c_2 . a_2^2} \right\} . \Delta t + 1 \right] . T^{n}_{2,i}$$

$$+ \left[\frac{k_2}{\Delta x} + \frac{\tau_2 . I}{2.a_2} \right] . \frac{\Delta t}{c_2 \Delta x} . T^{n}_{2,i-1} + \frac{\rho_{20} . I^2}{c_2 . a_2^2} . \Delta t \qquad \ldots(B.2)$$

The expression for the power input at x = 0 (cold junction), as derived in section 4.3.2 is :

$$P_0(t) = - \alpha_0 . T_0(t) . I(t) - k_1 . a_1 . \left. \frac{\partial T_1(x,t)}{\partial x} \right|_{x=0} - k_2 . a_2 . \left. \frac{\partial T_2(x,t)}{\partial x} \right|_{x=0} \qquad \ldots(4.16)$$

again discretising it we have :

$$P_0 = - \alpha_0 . T^{n+1}_{11} . I - k_1 . a_1 . \left(\frac{T^{n+1}_{12} - T^{n+1}_{11}}{\Delta x} \right) - k_2 . a_2 . \left(\frac{T^{n+1}_{22} - T^{n+1}_{21}}{\Delta x} \right)$$

where T_{12} denotes the temperature of the first element at the location specified by i = 2, and so on for the other terms.

Now the first boundary condition stipulates that the temperatures of the two elements are always equal at $x = 0$. Therefore :

$T_{11} = T_{21}$ for all times.

Hence the above expression can be rearranged as :

$$P_0 = \left[\frac{k_1.a_1}{\Delta x} - \alpha_0.I + \frac{k_2.a_2}{\Delta x} \right].T_{11}^{n+1} - \frac{k_1.a_1}{\Delta x}.T_{12}^{n+1} - \frac{k_2.a_2}{\Delta x}.T_{22}^{n+1} \qquad \text{...(B.3)}$$

The power output at $x = l$ (hot junction), as derived in section 4.3.2 is given by :

$$P_l(t) = \alpha_l.T_l(t).I(t) + k_1.a_1.\left.\frac{\partial T_1(x,t)}{\partial x}\right|_{x=l} + k_2.a_2.\left.\frac{\partial T_2(x,t)}{\partial x}\right|_{x=l} \qquad \text{...(4.17)}$$

discretising it we have :

$$P_l = \alpha_l.T_{2M}^{n+1}.I + k_1.a_1.\left(\frac{T_{1M}^{n+1} - T_{1M-1}^{n+1}}{\Delta x}\right) + k_2.a_2.\left(\frac{T_{2M}^{n+1} - T_{2M-1}^{n+1}}{\Delta x}\right)$$

where T_{1M} denotes the temperature of the first element at the point given by $x = l$, and similarly for the other terms.

From the boundary condition the temperatures of the two elements at $x = l$ must be equal for all times, so that

$T_{1M} = T_{2M}$

Hence the above expression can be rearranged as,

$$P_l = -\frac{k_1.a_1}{\Delta x}.T_{1M-1}^{n+1} - \frac{k_2.a_2}{\Delta x}.T_{2M-1}^{n+1} + \left[\alpha_l.I + \frac{k_1.a_1}{\Delta x} + \frac{k_2.a_2}{\Delta x} \right].T_{1M}^{n+1} \qquad \text{...(B.4)}$$

Equations B.1 to B.4 can be written in matrix form as :

$$
\begin{bmatrix}
\left[\dfrac{k_1 a_1}{\Delta x} - \alpha_0 I + \dfrac{k_2 a_2}{\Delta x}\right] & -\dfrac{k_1 a_1}{\Delta x} & 0 & 0 \cdots\cdots \dfrac{k_2 a_2}{\Delta x} & 0 & 0 \cdots\cdots\cdots\cdots 0 \\
0 & 0 & 0 & 0 \cdots\cdots -\dfrac{k_1 a_1}{\Delta x} & \left[\dfrac{k_1 a_1}{\Delta x} + \alpha_1 I + \dfrac{k_2 a_2}{\Delta x}\right] & -\dfrac{k_2 a_2}{\Delta x} \\
0 & 1 & 0 & 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 0 & & \\
0 & 0 & 1 & 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 0 & & \\
0 & 0 & 0 & 1 \quad 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 0 & & \\
\vdots & \vdots & \vdots & \vdots \quad \vdots \qquad \vdots \qquad \vdots & & \\
0 & 0 & 0 & 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 1 & &
\end{bmatrix}
\begin{bmatrix}
T_{11} \\ T_{12} \\ \vdots \\ T_{1M-1} \\ T_{1M} \\ T_{22} \\ \vdots \\ T_{2M-1}
\end{bmatrix}^{n+1}
$$

$$
= \begin{bmatrix}
P_0 \\
P_1 \\
-\left[\dfrac{k_1}{\Delta x} + \dfrac{\tau_1 I}{2a_1}\right]\dfrac{\Delta t}{c_1 \Delta x} T^n_{13} + \left[\left\{-\dfrac{2.k_1}{c_1 \Delta x^2} + \dfrac{\eta_1 . I^2}{c_1 . a_1^2}\right\}\Delta t + 1\right] T^n_{12} + \left[\dfrac{k_1}{\Delta x} - \dfrac{\tau_1 I}{2.a_1}\right]\dfrac{\Delta t}{c_1 \Delta x} T^n_{11} + \dfrac{\rho_{10} I^2}{c_1 a_1^2}\Delta t \\
\vdots \\
-\left[\dfrac{k_1}{\Delta x} + \dfrac{\tau_1 I}{2a_1}\right]\dfrac{\Delta t}{c_1 \Delta x} T^n_{1M+1} + \left[\left\{-\dfrac{2.k_1}{c_1 \Delta x^2} + \dfrac{\eta_1 . I^2}{c_1 . a_1^2}\right\}\Delta t + 1\right] T^n_{1M} + \left[\dfrac{k_1}{\Delta x} - \dfrac{\tau_1 I}{2.a_1}\right]\dfrac{\Delta t}{c_1 \Delta x} T^n_{1M-1} + \dfrac{\rho_{10} I^2}{c_1 a_1^2}\Delta t \\
\vdots \\
-\left[\dfrac{k_2}{\Delta x} - \dfrac{\tau_2 I}{2a_2}\right]\dfrac{\Delta t}{c_2 \Delta x} T^n_{2M+1} + \left[\left\{\dfrac{2.k_2}{c_2 \Delta x^2} - \dfrac{\eta_2 . I^2}{c_2 . a_2^2}\right\}\Delta t + 1\right] T^n_{2M} - \left[\dfrac{k_2}{\Delta x} + \dfrac{\tau_2 I}{2.a_2}\right]\dfrac{\Delta t}{c_2 \Delta x} T^n_{2M-1} - \dfrac{\rho_{20} I^2}{c_2 a_2^2}\Delta t
\end{bmatrix}
$$

Designating the three matrices as A, T and B respectively the above equation can be written in short form as :

$$A.T = B$$

The matrix T, ie. the temperature profile, is solved for on the computer. An example programme for the first case, ie. the response of the thermoelement for a step change in the current, is given in Appendix C. The programmes for the other cases are very similar to the one listed in Appendix C.

APPENDIX - C

PROGRAMME LISTING

```
c-----------------------------------------------------------------
c                                                                *
c       THIS PROGRAMME  CALCULATES THE  HOT AND  COLD  JUNCTION  *
c       TEMPERATURES BY SOLVING THE GENERAL GOVERNING EQUATIONS  *
c       FOR THE TEMPERATURE DISTRIBUTION  IN  A  THERMOELECTRIC  *
c       COOLING ELEMENT. THIS PROGRAMME  IS  FOR THE FIRST CASE  *
c       VIZ. A STEP CURRENT OF 0.5 A AND CONSTANT THERMAL POWER  *
c       INPUT OF 0.5 W.                                          *
c                                                                *
c-----------------------------------------------------------------
        Program  Dynamic_Analysis_Of_A_Thermo_Electric_Junction
c-----------------------------------------------------------------
c-----------------------------------------------------------------
        parameter (npara=200)
        Dimension x(npara), T(npara), v(npara,npara)
        Real i,i0,k1,k2,length
        common/volt/n,m,dt,w_old
        common/const/a10,a11,y1,y2,length,u1,u2,a1,a2,dx,p10,p20
c-----------------------------------------------------------------
c       OPENING THE INPUT DATA FILE "in".....
        open(11,file='in')
c-----------------------------------------------------------------
c       OPENING THE OUTPUT DATA FILE.....
        open(13,file='step0.5')
c-----------------------------------------------------------------
c       READING THE INPUT PARAMETERS FROM THE FILE "in".....
        read(11,*)p01
        read(11,*)dt,tmax
        read(11,*)n
        read(11,*)i0
        read(11,*)nitr
        m=2*n-2
c-----------------------------------------------------------------
```

```
c         ASSIGNING THE PROPERTY PARAMETERS OF THE THERMOELEMENT.....
          k1=2.4
          k2=2.2
          length=2.e-2
          u1= 0.91e-4
          u2= 0.91e-4
          w1=1.23e6
          w2=1.23e6
          y1=0.814e-5
          y2=0.814e-5
          p10=1.3e-5
          p20=1.3e-5
          a1=0.836e-4
          a2=0.836e-4
          a10=2.3e-4
          a11=2.3e-4
          dx=length/m
          T0=25.0
          p0=0.
          p1=0.
          w_old=0.
c-----------------------------------------------------------------
c         INITIALIZING FOR TEMPERATURE.....
          do j=1,(m+1)
          T(j)=T0
          enddo
c-----------------------------------------------------------------
          time=0
          itr=0
          i=0.
          write(13,*)'Temp. profile for a step current of 0.5 amperes'
          write(13,*)'     '
          write(13,*)'         TIME         HOT JUNC.      COLD JUNC. '
1000      itr=itr+1
          time=time+dt
```

```
      if (time.ge.5) go to 2
      go to 3
2     i=i0
      p0=p01
      p1=-p01 + volt*i +w_rate
3     continue
c----------------------------------------------------------------------
c     CALCULATING  THE  COEFFICIENTS  OF  THE  DISCRETIZED
c     EQUATIONS.....
c
c     COEFFICIENTS OF EQUATION B.1
      b1 = ((k1/dx)+(u1*i)/(2*a1))*(dt/(w1*dx))
      b2 = ((-2*k1)/(w1*dx**2)+(y1*i**2)/(w1*a1**2))*dt +1.
      b3 = ((k1/dx)-(u1*i)/(2*a1))*dt/(w1*dx)
      b4 = ((p10*(i**2)*dt)/(w1*a1**2))
c----------------------------------------------------------------------
c     COEFFICIENTS OF EQUATION B.2
      d1 = ((k2/dx)-(u2*i)/(2*a2))*dt/(w2*dx)
      d2 = ((-2*k2)/(w2*dx**2)+(y2*i**2)/(w2*a2**2))*dt +1.
      d3 = ((k2/dx)+(u2*i)/(2*a2))*dt/(w2*dx)
      d4 = ((p20*(i**2)*dt)/(w2*a2**2))
c----------------------------------------------------------------------
c     COEFFICIENTS OF EQUATION B.3
      c1 = ((k1*a1)/dx-(a10*i)+(k2*a2)/dx)
      c2 = -((k1*a1)/dx)
      c3 = -((k2*a2)/dx)
c----------------------------------------------------------------------
c     COEFFICIENTS OF EQUATION B.4
      e1 = -((k2*a2)/dx)
      e2 = (((k1*a1)/dx)+(a11*i)+((k2*a2)/dx))
      e3 = -((k1*a1)/dx)
c----------------------------------------------------------------------
c     ASSIGNING THE TERMS TO THE R.H.S. MATRIX.....
      x(1)=p0
c----------------------------------------------------------------------
```

```
        do j=2,n-1
        x(j)=b1*T(j+1)+b2*T(j)+b3*T(j-1)+b4
        enddo
c------------------------------------------------------------------
        x(n)=p1
c------------------------------------------------------------------
        do j=n+1,m
        x(j)=d1*T(j+1)+d2*T(j)+d3*T(j-1)+d4
        enddo
c------------------------------------------------------------------
c       ASSIGNING THE TERMS TO THE COEFFICIENT MATRIX.....
        do j=1,m
        do k=1,m
        v(j,k)=0.0
        enddo
        enddo
c------------------------------------------------------------------
        v(1,1)=c1
        v(1,2)=c2
        v(1,n+1)=c3
        v(n,m)=e1
        v(n,n)=e2
        v(n,n-1)=e3
c------------------------------------------------------------------
        do j=2,n-1
        v(j,j)=1.0
        enddo
c------------------------------------------------------------------
        do j=n+1,m
        v(j,j)=1.0
        enddo
c------------------------------------------------------------------
        call gass(v,x,T,m)
        call voltage(i,t,volt,w_rate)
        if( mod(itr,nitr) .eq. 0) then
```

```
      write(13,222) time,T(1),T(n)
      print*,time,T(1),T(n)
      endif
      if( time .lt. tmax) go to 1000
1     continue
      rewind 13
222   format(3f15.5)
      stop
      end
c-----------------------------------------------------------------------
c     THIS SUBROUTINE CALCULATES THE VOLTAGE BY SOLVING EQUATION      *
c     4.25, USING THE TEMPERATURE VALUES CALCULATED IN THE            *
c     PROGRAMME.                                                      *
c                                                                     *
c-----------------------------------------------------------------------
      subroutine VOLTAGE(i,t,volt,w_rate)
      parameter (npara=200)
      Dimension  T(npara)
      common/volt/n,m,dt,w_old
      common/const/al0,al1,y1,y2,length,u1,u2,a1,a2,dx,p10,p20
      real i,length
      sum1=0.0
      sum2=0.0
      do k=1,n-1
      tav=0.5*(t(k)+t(k+1))
      sum1=sum1+tav*dx
      enddo
      do k=n+1,m
      sum2=sum2+ t(k)*dx
      enddo
      sum2=sum2+ 0.5*dx*(t(n)+t(1))
      w_new=a1*sum1 + a2*sum2
      w_rate=(w_new-w_old)/dt
      w_old=w_new
      term1= al1*t(n)-al0*t(1)
```

```
      term2= -i*(length) * (p10/a1+p20/a2)
      term3= -i*y1*sum1/a1
      term4= i*y2*sum2/a2
      volt=term1+term2+term3+term4
      return
      end
c------------------------------------------------------------------
c     THIS SUBROUTINE SOLVES FOR THE TEMPERATURE DISTRIBUTION      *
c     IN THE ELEMENT  BY  USING THE  GAUSS-JORDAN ELIMINATION      *
c     TECHNIQUE.                                                   *
c------------------------------------------------------------------
      SUBROUTINE GASS(A,Y,XX,N)
      parameter (npara=200)
      DIMENSION A(npara,npara),Y(npara),XX(npara)
      M=N-1
      DO 10 I=1,M
      L=I+1
      DO 10 J=L,N
      IF(A(J,I))6,10,6
6     DO 8 K=L,N
8     A(J,K)=A(J,K)-A(I,K)*A(J,I)/A(I,I)
      Y(J)=Y(J)-Y(I)*A(J,I)/A(I,I)
10    CONTINUE
      XX(N)=Y(N)/A(N,N)
      DO 30 I=1,M
      K=N-I
      L=K+1
      DO 20 J=L,N
20    Y(K)=Y(K)-XX(J)*A(K,J)
      XX(K)=Y(K)/A(K,K)
30    CONTINUE
      RETURN
      END
c------------------------------------------------------------------
c------------------------------------------------------------------
```